BANK OF ENGLAND

Staff Working Paper No. 814

Real effects of financial distress: the role of heterogeneity

Francisco Buera and Sudipto Karmakar

October 2021

This is an updated version of the Staff Working Paper originally published on 2 August 2019

BANK OF ENGLAND

Staff Working Paper No. 814

Real effects of financial distress: the role of heterogeneity

Francisco Buera[(1)] and Sudipto Karmakar[(2)]

Abstract

Which firms are more sensitive to an aggregate financial shock? What can be learnt from these heterogeneous responses? We evaluate and answer these questions from both empirical and theoretical perspectives. Using micro data from Portugal during the sovereign debt crisis we find that highly leveraged firms and firms with a larger share of short-term debt on their balance sheets contracted more in the aftermath of the financial shock. We analyse the conditions under which leverage and debt maturity determine the sensitivity of firms' investment decisions to financial shocks in standard models of investment under financial frictions. In doing so, we extend these models to feature a maturity choice. We show that simple versions of these models are not consistent with the observed heterogeneous responses. The model needs the presence of frictions when issuing long-term debt to rationalise the empirical findings.

Key words: Sovereign debt, leverage, maturity structure.

JEL classification: E44, F34, G12, H63.

(1) Washington University in St. Louis. Email: fjbuera@wustl.edu
(2) Bank of England. Email: sudipto.karmakar@bankofengland.co.uk

We want to thank Manuel Adelino, Martin Arazi, Jeff Campbell, Leonardo Gambacorta, Simon Gilchrist, Francois Gourio, Robert Hall, Narayana Kocherlakota, Ramon Marimon, Alberto Martin, Ellen McGrattan, Emi Nakamura, Derek Neal, Juan Pablo Nicolini, Steven Ongena, Daniel Paravisini, Glenn Schepens, Petr Sedlacek, David Sraer, Judit Temesvary and seminar/conference participants at the NBER Monetary Economics Meeting (Spring 2018), Bank of England, NYU, LSE, UBC, Oxford, York, BIS, European Economic Association, Bank of Portugal, Bank of Hungary, FRB Chicago, Kings College, Nottingham, Glasgow, SED Annual Meetings 2017, ESCB Day Ahead Conference 2017, and the ECB-Bank of Greece Conference on Macroprudential Policy for helpful comments and discussions. Sudipto Karmakar also wishes to acknowledge the support of the BIS, as part of the project was completed while he was visiting as a Research Fellow. The views expressed are those of the authors and do not reflect the views of the Bank of England or its policy committees, the Bank of Portugal or the Eurosystem.

The Bank's working paper series can be found at www.bankofengland.co.uk/working-paper/staff-working-papers

Bank of England, Threadneedle Street, London, EC2R 8AH
Email publications@bankofengland.co.uk

ISSN 1749-9135 (on-line)

1 Introduction

Increasingly, studies on macro-finance are using quasi-experimental data to identify aggregate financial shocks to firms. These studies then use rich micro data to trace their real consequences (e.g., Chodorow-Reich, 2014; Acharya et al., 2018). Further, a long empirical tradition exists that studies the heterogeneous responses of firms to aggregate shocks to understand which firms are more fragile, or as an alternative strategy to identify these shocks (e.g., Gertler and Gilchrist, 1994; Almeida et al., 2012). Notwithstanding the wealth of new and rich data, there are limits to a purely empirical strategy. These limitations are more salient when analysing heterogeneous responses. The heterogeneous characteristics of firms are likely to be endogenous and not under the control of the experimenter. A natural question then is what can be learnt from the heterogeneous responses of firms to financial shocks? More specifically, we wish to understand incorporating which frictions in macro models might help us rationalise the heterogeneous responses of firms to financial shocks. We study such responses both in the data and in a class of environments common in the macro-finance literature to answer this question.

This study makes an empirical and a theoretical contribution. First, using detailed micro data from Portugal around the 2010-2012 sovereign debt crisis, we find that highly leveraged firms and firms with a larger share of short-term debt on their balance sheets contracted more in the aftermath of the financial shock. We also show that the heterogeneity in the responses to the financial shock are mainly driven by differences in the costs of funds across firms. Second, we characterise the differential responses by using a suite of theoretical models that feature financial frictions as commonly modelled in the macro-finance literature. In doing so, we extend these models to feature a maturity choice. We show that these models are not consistent with the differential responses found in the data, unless they incorporate an additional friction to the choice of maturity. Our empirical and theoretical results taken together imply that negative bank shocks disproportionately affect real outcomes of firms exposed to rollover risk which in turn is driven by the high cost of funds for these firms.

For the empirical analysis, we use Portugal as a laboratory because it is a country that suffered a large financial shock while the sovereign debt crisis was unfolding in Europe.[1] We measure the financial shock as the interaction between the sovereign debt crisis and the pre-crisis Portuguese sovereign debt holdings of the banks (as a fraction of their total assets) from which individual firms borrowed.[2] Reasonably, the net worth of banks with large holdings of sovereign debt deteriorates more when the crisis occurs, as the market value of the sovereign bonds on their balance sheets declines sharply. To the extent that the net worth of banks determines their supply of loans, the credit supply to individual firms also contracts.

[1]The magnitude of the sovereign debt crisis in Portugal, as measured by the rise in the sovereign risk premium, is second only to Greece, a country for which there is no data available of similar quality.

[2]Our strategy to measure financial shocks follows recent contributions using credit registry and balance sheet information of firms and banks (Chodorow-Reich, 2014; Acharya et al., 2018).

Importantly, we find that this effect is more pronounced for firms with high leverage and short-term debt. We show that a bank in the 90^{th} percentile of sovereign holdings cuts lending to a highly leveraged firm by 3.15 percentage points more than a bank in the 10^{th} percentile. The differential cut in credit is 4.3 percentage points for firms categorised as having high short-term debt. After quantifying the bank lending channel, we analyse real outcomes. We show that highly leveraged firms and firms with a larger share of short-term debt contracted significantly more in the immediate aftermath of the sovereign debt crisis, compared to their counterparts. The variables we analyse are employment, investment, total debt, and the usage of intermediate commodities. We ensure the validity of our results by conducting an array of robustness exercises.

On the theoretical side, we analyse the conditions under which leverage and debt maturity determine the sensitivity of firms' investments to financial shocks. While the interpretation of the aggregate shock is relatively straightforward, it is not so for the heterogeneous responses. Firm characteristics, for example, leverage and maturity, are not randomly assigned and, therefore, we require a model to interpret these findings. In particular, under which assumptions can we rationalise these results or, in other words, what lessons can we draw about relevant model elements from these results? We show in our theoretical analysis that the presence of long-term investments and frictions in the issuance of long-term debt, as captured by a firm-specific term premium, are needed for the model to generate similar heterogeneous responses of investment to the financial shock.

In the benchmark analysis, we consider a standard model of entrepreneurs who face a linear investment opportunity subject to uninsurable idiosyncratic investment risk and shocks to the interest rate to interpret our empirical results. The key friction in the model is the inability of entrepreneurs to insure against idiosyncratic shocks, which is a common assumption in the recent macro-finance literature (Brunnermeier and Sannikov, 2014; Arellano et al., 2016). The model is enriched to feature heterogeneous cash flows from an initial long-term investment project and an initial choice of debt maturity. We analyse the conditions under which initial leverage and debt maturity determine the sensitivity of firms' investments to the financial shock.

We first consider the case in which the initial leverage and the debt maturity are exogenous. In this case, we show that the sensitivity of firms' investment to the interest shock is an increasing function of leverage if the future net cash flows from the initial investment project are positive. In turn, the sensitivity of firms' investment to the interest shock is an unambiguously decreasing function of the maturity of the debt. This part of the analysis is broadly consistent with the conventional view.

We then analyse the case in which the debt maturity is endogenous. We consider situations in which the variation in the maturity of debt reflects heterogeneity across entrepreneurs in the timing of the cash flows from the initial long-term investment project and in the term premium faced by these entrepreneurs. Furthermore, we assume that in the empirical analysis we cannot perfectly control for determinants of the maturity structure, so that there is residual cross-sectional variation in the maturity of debt. We show that if the (residual) variation reflects heterogeneity across entrepreneurs in the timing of the cash flows, then the sensitivity of firms' investment to the interest rate shock is independent of leverage and debt maturity. Only when the variation in the maturity of debt reflects heterogeneity in the term premium does the model reproduce our empirical results.

To see the intuition of this result, consider two entrepreneurs with different debt maturities but the same amounts of leverage. Furthermore, assume that the differences between these two entrepreneurs are driven by differences in the expected short-term cash flows from the long-term projects. In this case, the entrepreneur with the shorter maturity has larger expected short-run cash flows so that a shorter maturity structure is optimal. In other words, when considering entrepreneurs with shorter debt maturities but the same amounts of leverage, we are (positively) selecting entrepreneurs with higher short-term cash flows. In this example, the two entrepreneurs have exactly the same financial needs. This implies that the sensitivity of investment to the financial shock remains unchanged. The fact that the debt maturity is optimally chosen is key for the positive selection effect to cancel the negative direct effect of leverage. A similar reasoning applies when we consider entrepreneurs with more leverage, but with the same debt maturities. In the case in which the heterogeneity in debt maturity is driven by differences in the term premium, the selection effect goes in the same direction as the direct effect. In this case, the entrepreneur with the shorter maturity has a higher cost of long-term debt and, therefore, larger financial needs. This implies that the sensitivity of her investment to the interest rate shock is even higher.

We explore the sensitivity of the results to various extensions and modifications of the benchmark model: (a) a richer specification in which the aggregate shock also affects the revenue of entrepreneurs in the interim period; (b) an alternative model that features risk-neutrality, diminishing returns, and collateral constraints, which are another common set of assumptions in the macro-finance literature; (c) a version of the model in which the initial leverage is endogenously determined by firms' productivity and initial resources in the first period. The main results either generalise or get strengthened with these alternative assumptions.

Related Literature Our work relates most closely to the recent empirical literature that uses micro-data to identify and measure the effects of financial shocks and a theoretical macro-finance literature that proposes alternative models of the links between the financial and real sectors.

Chodorow-Reich (2014) shows that the firms that had pre-crisis relationships with banks that struggled during the crisis reduced employment more than the firms that had relationships with healthier lenders. Similar findings are reflected in Bentolila et al. (2018). Iyer et al. (2014) and Cingano et al. (2016) show how disruptions in the interbank market can have adverse real consequences. Closer to our focus on the European sovereign debt crisis, Bofondi et al. (2018) use data from the Italian credit register to study the aggregate effects of the sovereign debt crisis on the credit supply. Bottero et al. (2015) also use similar data to show that financial intermediaries passed on the exogenous shock to their sovereign securities to firms through a contraction of the credit supply. Acharya et al. (2018) explore the impact of the European sovereign debt crisis and the resulting credit crunch on the corporate policies of firms, using data from the syndicated loan market. This literature is, primarily, based on the *ex ante* heterogeneity of lenders.

We take the analysis further by focusing on how the heterogeneities of lenders and borrowers interact. When banks are in distress, they do not affect all borrowers in a uniform manner. We identify important dimensions of firm fragility (i.e., leverage and the debt maturity) and quantify the effects along the entire distribution of these variables. Exploring the heterogeneous interactions between lenders and borrowers is quite novel and only recently has an emerging strand of literature attempted to do so. Almeida et al. (2012) use "long-term debt maturity [...] as an identification tool" to measure the causal effect of the 2007 financial crisis on investment. Benmelech et al. (2018) also use pre-existing variation in the values of long-term debt that came due during a crisis episode to identify a financial shock. Giroud and Mueller (2017) show that more highly leveraged firms experienced significantly larger employment losses in response to a decline in local consumer demand.[3]

In recent work, Kalemli-Ozcan et al. (2020) quantify the role of financial leverage and debt maturity behind the sluggish investment performance of European firms in the 2008-2012 crisis period. To do this, they use panel data on firms from 2000-2012 and data on bank-firm relationships after 2013. Our work focuses on the effect of the 2010 sovereign debt crisis and uses detailed information on bank-firm relationships prior to the debt crisis. In addition, we analyse in a theoretical model the conditions for leverage and debt maturity to interact with the sensitivity of firms' investment decisions to interest rate shocks.

We contribute to the theoretical macro-finance literature by analysing alternative models of the links between the financial and real sectors and shed light on the model elements that are important to capture the observed effects of financial shocks.[4] The macro-finance literature has used alternative models of financial frictions and specifications of the investment technologies. For instance, the inability of entrepreneurs to insure against idiosyncratic shocks is a common assumption, e.g., Angeletos (2007); Brunnermeier and Sannikov (2014); Arellano et al. (2016). Collateral constraints are another popular device to introduce financial frictions into macro models, for example, Kiyotaki and Moore (1997) and Holmstrom and Tirole (1997). While constant returns are a convenient modelling choice, diminishing returns have been featured in quantitative-oriented analyses of financial shocks, for example, Khan and Thomas (2013), Buera et al. (2015) among others. Our empirical and theoretical results point out important model elements that are often

[3]Related evidence is presented by De-Socio and Sette (2018) for Italian firms in the aftermath of the global financial and sovereign debt crises. In a more structural setting, Arellano et al. (2017) use the heterogeneous responses of firms to calibrate by how much a rise in the sovereign premium affects a firm's interest rate. They argue that "the implications of these higher borrowing rates are not homogeneous in the population of firms, because they are more damaging to the performance of firms with large borrowing needs," that is, more leverage.

[4]In this respect, we follow the early lead by Gertler and Gilchrist (1994) who study the response of small versus large manufacturing firms to monetary policy in order to obtain evidence on the importance of financial propagation mechanisms for aggregate activity. See Crouzet and Mehrotra (2018) for a recent evaluation of this strategy.

absent in the theoretical analysis. We find that the presence of long-term investment projects and frictions to the issuance of long-term debt, as captured by a firm-specific term premium, are needed for the model to account for the heterogeneous response of investment to the financial shock in our empirical analysis. Thus, through the light of the theory, our empirical results highlight the importance of these model elements to understand the real effects of financial shocks.

We proceed as follows: Section 2 provides a brief overview of the macroeconomic events in the lead up to the sovereign debt crisis. Section 3 provides our main empirical analysis. We start by describing the data and lay special emphasis on showing the *absence* of adverse firm-bank matching in the data. Next, we proceed to the lending and real effects regressions. Section 4 presents our theoretical analysis. In Section 5, we study the empirical determinants of debt maturity under the light of the implications of the theoretical model, and Section 6 concludes. All figures, tables, and proofs of the propositions are in the appendix.

2 Events Preceding the Crisis

Until late 2009 or early 2010 the viability of sovereign debt was not a concern for the markets. However, in the spring of 2010, when the Greek government requested an EU/IMF bailout package to cover its financial needs for the remaining part of the year, markets started to doubt the sustainability of sovereign debt. Soon after Standard & Poors downgraded Greece's sovereign debt rating to BB+ ("junk bond") that led investors to be concerned about the solvency and liquidity of the public debt issued by other peripheral Eurozone countries like Ireland and Portugal.

In May 2010, following the Greek bailout request, the CDS spreads on Portuguese sovereign bonds increased dramatically (Figure 1, left panel) and suddenly the Portuguese banks lost access to international debt markets. The central panel of Figure 1 shows the decline in funding obtained through securities (market funding) as a fraction of bank liabilities. The banks could not obtain funding in medium and long-term wholesale debt markets, and these had been an important source of their funding until then. This sudden stop is attributed mainly to investors' concerns about contagion from the sovereign crisis in Greece. This sudden rise in CDS spreads and the subsequent loss of access to market funding meant that the banks that were more exposed to the public sector saw the risk in their balance sheets going up.[5] The banks passed on the increase in their cost of funding to their borrowers. The right panel of Figure 1 shows the evolution of the spread between the average lending rates by banks at one year maturity relative to the return of a 1-year German sovereign bond. The extreme left and right panels lend credence to the fact that the sovereign and lending rates are closely related. We call this channel of transmission of a shock as the *sovereign channel*.[6]

[5] Fears about the solvency of the sovereign can put the solvency of banks at risk, since banks typically hold a substantial portion of their assets in the form of sovereign debt (Brunnermeier et al., 2011).

[6] The sovereign CDS spread information is from Bloomberg (Bloomberg L.P., 2021). The bank lending spreads to Portuguese and other Eurozone countries non-financial corporations uses information from the Euro area and national

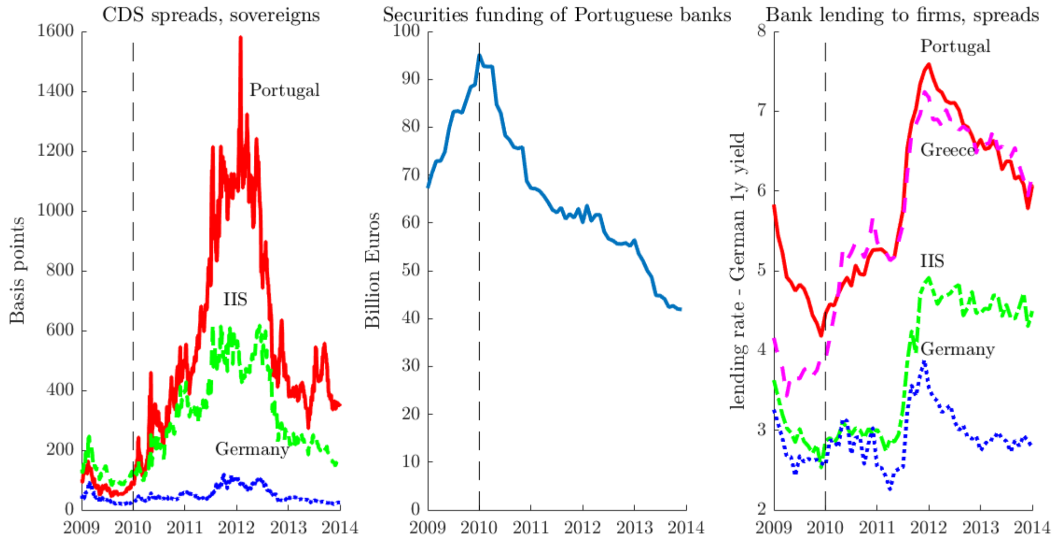


Figure 1: Evolution of sovereign CDS spreads, market funding to Portuguese banks, and bank lending spreads to Portuguese and other Eurozone countries non-financial corporations during the sovereign debt crisis.

3 The Empirical Analysis

3.1 Data description

We build a comprehensive and unique dataset for the Portuguese economy by combining three separate datasets. The datasets are the Central Credit Register (CRC), the Central Balance Sheet Database (CBSD), the Bank Balance Sheet Database (BB). The Bank of Portugal manages the CRC that contains information reported by the participants (the institutions that extend credit) about the credit granted to nonfinancial corporations. The CRC records any loan of 50 Euros or more. The CBSD is based on accounting data of individual firms. The sample includes all firms that file taxes in Portugal. The BB database provides information on the components of balance sheets such as banks' sovereign exposures, size, capital ratios, and liquidity ratios.[7]

In Tables 1, 2, and 3, we provide an overview of our dataset. All figures correspond to 2009:Q4. Table 1 presents the aggregate statistics on firms. Column 1 represents all firms from the CBSD. Column 2 shows the subset of firms that have also obtained credit while the last column shows only firms that have multiple banking relationships. To improve the quality of the analysis, we drop the micro firms, that is, all firms that had an outstanding loan amount less than 10,000 Euros as of the last quarter of 2009. The lower panel of Table 1 gives the data presented in the top right panel. We present firm characteristics based on the heterogeneity of leverages and debt maturities,

MFI interest rates (MIR) (European Central Bank, 2016), together with data on the return on the 1-year German Treasury Bill (GTDEM1Y) from Bloomberg (Bloomberg L.P., 2021).

[7]See Banco de Portugal (2019a), Banco de Portugal (2019b) and Banco de Portugal (2019c).

since these are the two dimensions we study in this paper. A highly leveraged firm is one that has more than 47% leverage while a firm with high short-term debt is one with more than 53%, where both these numbers correspond to the 75th percentiles of the respective distributions. Our final sample of firms is quite representative of the Portuguese economy. The sample represents 71% of the total loans granted as of December 2009. It further represents 70.51% of the aggregate employment in Portugal, 76.41% by turnover, and 77.07% by assets. Further, we check if the labour share of each sector in the population closely matches the labour share of each sector in the sample. The correlation coefficient stands at 0.98, with the three largest sectors by employment being manufacturing, wholesale and retail services, and construction.

Before proceeding any further in our empirical investigation, we need to verify that firms and banks were not matched (*ex ante*) in an adverse, observable manner. In other words, were (*ex post*) weak banks lending to weak firms prior to the crisis? To see that this is not the case, we need to show that the banks that were differently exposed to the sovereign (*ex ante*) were not operating different business models, did not have different funding structures, or most importantly, did not have different types of client profiles. In Table 2, we provide detailed bank characteristics of 33 banking groups in Portugal. Lending to the non-financial corporations appears to be a central part of the business of banks in Portugal. Banks that are more exposed to the sovereign have higher liquidity ratios and lower exposures to the household sector. In terms of the corporate exposure, the two groups are very comparable. We also compare the funding structures of the banks, namely, security funding, inter-bank market borrowing, and central bank funding. We find no notable difference between the two groups and none of the differences are statistically significant, as shown in the last column.

Table 3 presents the weighted borrower characteristics of high and low sovereign banks. Once again, we find no significant difference between the two groups. Besides the tables, Figure 9 in the Online Appendix plots the banks' sovereign exposures versus their shares of nonperforming loans for four quarters prior to the shock. These correlations turn out to be negative and insignificant that further confirm the fact that the banks that were holding more public debt did not necessarily have more risky balance sheets, *ex ante*.[8]

3.2 Regression analysis

For the empirical analysis, all growth rates are constructed by following Davis and Haltiwanger (1992):

$$g_t^E = \frac{E_t - E_{t-1}}{x_t},$$

[8]See subsection B7 in the Online Appendix.

in which g_t^E is the growth rate of variable 'E' at time 't' and x_t is the mean of the variable over the current and the last period; that is,

$$x_t = 0.5 * (E_t + E_{t-1}).$$

This measure of net growth is bounded between +2 and -2 and symmetric around zero.[9]

We now proceed in two steps. We first analyse the effects on the credit supply during the crisis and then study the real effects of the sovereign debt crisis. To identify the effects on lending, we use the method developed in Khwaja and Mian (2008). Around two thirds of the firms in our sample have multiple banking relationships, and we exploit this fact to identify if there were any adverse effects on lending. The first equation we estimate is the following:

$$\begin{aligned} g^L_{i,b,Q4:10-Q4:09} &= \beta_0 + \beta_1 \text{sov\_share}_{b,Q4:09} + \beta_2 \text{sov\_share}_{b,Q4:09} * D^{lev} + \beta_3 D^{lev} \\ &+ \beta_4 \text{sov\_share}_{b,Q4:09} * D^{stdebt} + \beta_5 D^{stdebt} + \Gamma_b B_{b,Q4:09} + \gamma_i + \epsilon_{i,b}, \end{aligned} \tag{1}$$

in which $g^L_{i,b,Q4:10-Q4:09}$ is the growth rate of total committed credit between a firm-bank pair (i, b) between Q4:09 and Q4:10, $\text{sov\_share}_{b,Q4:09}$ is the sovereign share of bank 'b' in Q4:09, γ_i is a vector of firm fixed effects that help us control for demand side factors, and $B_{b,Q4:09}$ is a vector of bank-level controls prior to the shock. We also include interaction terms with high leverage and high short-term debt dummies to identify the heterogeneous effects in the data. The variable D^{lev} is a dummy that identifies firms in the top quartile of leverage. A firm is considered to be highly leveraged if it reports a pre-crisis leverage of greater than 47%. A firm with a share of short-term debt greater than 53% is categorised as a high short-term debt firm, and D^{stdebt} is a dummy that helps us identify these firms.

The results are presented in Table 4. Columns 1 - 5 give regression results for firms having multiple banking relationships, while columns 6 - 8 include firms having single banking relationships as well, for the sake of completeness. Columns 1 - 2 present the baseline case for the interactions of the sovereign exposure with the high leverage and high short-term debt dummies. Columns 3 - 8 saturate the regressions with an array of bank level controls.[10] Overall, we find that there was a statistically significant reduction in lending to firms that were highly leveraged and those that had a significant share of short-term debt on their balance sheets. In terms of economic magnitudes, these effects are quite substantial as well. For the highly leveraged firms, the bank with a sovereign exposure in the 90th percentile reduces lending by 3.15 percentage points more than a bank in the

[9] A value of +2 corresponds to entrants while a value of -2 corresponds to firms that exit the market. This method of computing the growth rates helps us account for both the intensive and extensive margins and also helps us minimise the effects of outliers. This method of computing the growth rate is monotonically related to the conventional measure and the two are equal for small growth rates in absolute value. Further, if G_t^E is the conventional growth rate measure, that is, the change in a variable normalised by the lagged value of that variable, then $G_t^E = 2g_t^E/(2 - g_t^E)$.

[10] In unreported regressions, we verify that our results are completely robust to the inclusion of bank fixed effects and lead bank fixed effects.

10th percentile to the same firm. The same figure stands at 4.3 percentage points for firms with a high share of short-term debt on their balance sheets. Besides interacting the bank sovereign exposures with a dummy corresponding to the top quartile of leverage and short-term debt, we also do so with dummy variables for firms in all four quartiles to study the credit supply effects on firms belonging to each of these quartiles. Figure 3 illustrates these results. We observe how the credit contraction is much more pronounced for firms in the top-most quartile of leverage and short-term debt, when compared to firms in the lower quartiles.

For the analysis of real effects, we construct a weighted sovereign exposure measure for each firm. We note all the bank-firm relationships in the Q4:2009 and banks' sovereign holdings as a fraction of their total assets. Using the relative shares of each bank in a firm's loan portfolio, we construct a measure of sovereign exposure for each firm. For the rest of the analysis we keep the shares, and therefore, exposures constant. In other words, a firm's exposure to the sovereign through its lenders is predetermined in our model. Therefore, our measure ($sov_{i,\text{Q4:2009}}$) is calculated as:

$$sov_{i,\text{Q4:2009}} = \sum_{b \epsilon B} s_{i,b} * \text{sov\_share}_b,$$

in which $s_{i,b}$ is the share of bank 'b' in the total borrowing of firm 'i', and sov_share_b is the total Portuguese sovereign bond holdings of bank 'b' normalised by total assets. Figure 2 presents the distribution of the weighted sovereign exposures of the firms in the fourth quarter of 2009. The important implicit assumption is that the banks transmit shocks to the real sector that are proportional to their pre-crisis lending relationships. To verify the validity of this assumption, we show that firm-bank relationships are extremely persistent in Portugal. The probability of a firm-bank relationship continuing in the next period, conditional on it existing in the current period, is around 0.87. The probability that a bank remains a firm's *lead* lender in the next period, conditional on it being the *lead* lender in the current period, is 0.80. The lead lender of a firm is the bank with which it has the largest loan balance. Furthermore, the persistence of past relationships do not decline but actually increase slightly during the sovereign crisis. Table A1 in the Online Appendix presents these results.

The real variables we use in our analysis are firms' employment, fixed assets, total debt, and the usage of intermediate commodities.[11] To construct the growth rates, we use stocks in the fourth quarter of 2009 and 2010. The regression equation we estimate is given as:

[11]The rationale for including a broad measure of total debt (including trade credit, bond issuances, etc) as one of our real variables must be clarified. By means of estimating equation (1), we show that fragile firms experienced a sharper decline in credit supply. A natural question that arises is whether they were able to substitute the loss in funding by moving to other less exposed banks or by taking recourse with other forms of funding such as trade credit or bond markets? This was indeed not the case. If it were, we would not observe a decline in total debt that is a comprehensive measure of all firm liabilities. Therefore, our total debt measure helps us show that these fragile firms were not able to instantaneously seek funding elsewhere. This is not surprising to the extent that bank funding is the primary source of external financing, and less than 1 percent of Portuguese firms issue bonds.

$$
\begin{aligned}
g^{V}_{i,Q4:10-Q4:09} &= \alpha_0 + \alpha_1 sov_{i,Q4:09} + \alpha_2 sov_{i,Q4:09} * D^{lev} + \alpha_3 D^{lev} \\
&+ \alpha_4 sov_{i,Q4:09} * D^{stdebt} + \alpha_5 D^{stdebt} + \Gamma^1_i F_i + \Gamma^2_b B_b + \gamma^{ind} + \gamma^{loc} + \epsilon_i, \qquad (2)
\end{aligned}
$$

in which the superscript 'V' represents employment, fixed assets, total debt, and intermediate commodities; and $sov_{i,Q4:09}$ represents the weighted sovereign holdings of the firm in Q4:2009. As before, D^{lev} and D^{stdebt} help identify firms in the top quartile of leverage and short-term debt, respectively. The vector F_i contains firm-specific controls that comprise measures of profitability, age, size, leverage, and the maturity structure of debt, depending on the specification. The vector B_b comprises the weighted bank controls and the variables we use here are the bank's average size, average interest rate on loans, capital ratio, and the liquidity ratio. We also add industry (γ^{ind}) and location (γ^{loc}) fixed effects to our regressions, following our discussion of firm-bank matching earlier.

The results are reported in Table 5. The coefficient for the sovereign share variable captures the impact for the low leveraged firms and firms with a greater share of long-term debt. We do not find any statistically significant effects for these groups of firms. The total real effect of the crisis on the highly leveraged firms can be obtained by taking the sum of the coefficients for the sovereign exposure term and the interaction term in columns 1 - 4. The same is done for firms with a high amount of short-term debt in columns 5 - 8. In columns 9 - 12, we include both interactions for the sake of completeness. For the subcategory of the highly leveraged firms and firms with high short-term debt, we find that the employment, capital, total debt, and intermediate commodities all show a sizable decline. The economic magnitudes are also quite significant. For the highly leveraged firms, moving from the 10th percentile of the distribution of weighted sovereign exposures to the 90th percentile, we observe a decline of 1.7% in terms of employment relative to their less leveraged counterparts. The contractions in terms of assets, total debt, and intermediate commodities were 7.2%, 13.8%, and 3.9% respectively.

The results on the maturity structure are presented in columns 5 - 8. As in the previous case, we find statistically significant and negative effects on the firms that have a larger share of short-term debt on their balance sheets. These results are robust across all our real variables. For a firm with a higher share of short-term debt, moving from the 10th to the 90th percentile of weighted sovereign exposures brings about a fall of 1.2% in terms of employment, 2.3% in terms of assets, 2.5% in terms of total debt, and 1.9% in terms of intermediate commodity usage. Columns 9 - 12 broadly show that both dimensions of heterogeneity are equally important to understanding the real effects on the corporate sector.

It might also be interesting to study the effects along the entire distribution of leverage and the maturity of debt. Figure 4 presents the impact on our outcome variables. This is done separately by grouping firms into four leverage and maturity bins (by quartiles). Panel (a) gives the results for leverage, while panel (b) gives the results for short-term debt. In the regression analysis presented earlier, we compared the top quartile with the bottom three quartiles. This analysis breaks it down further to shed light on how firms in each of these quartiles perform in the immediate aftermath of the sovereign debt crisis and to uncover potential nonlinearities in the data. We observe that as we move from the lowest to the highest quartile of leverage and debt maturity, the firms were more adversely affected. In other words, the effects are much more subdued for firms with the lowest leverage and those firms with a longer maturity of debt, when compared to their counterparts in the upper quartiles.

Discussion of Robustness Exercises To ensure the stability and validity of our results, we conduct an array of robustness exercises. These results can be found in the Online Appendix B, and some of the main ones are briefly discussed here. First, we demonstrate the parallel trends to convince the reader that the effect we have found is not confounded by other factors. We also trace the effects over time. The latter part is purely for illustrative purposes as a number of actions taken by the governments and central banks make identification particularly difficult. Second, we ensure that the results are robust to the holdings of sovereign debt from other southern European countries (Spain, Italy, Greece, and Ireland). Third, we verify that our results are robust to specifying the time windows differently around the incidence of the shock. Fourth, we show that our results are not driven by any particularly vulnerable sector that could be directly hit by the shock. We argue that a potential candidate could be the construction sector which could have exposure to the public sector. We show that our results are unaltered if we drop this sector from our analysis. Fifth, we verify that the presence of foreign banks does not influence our analysis as branches of foreign banks could be bailed out by the parent entity. Sixth, we verify that there was no adverse matching between weak firms and banks prior to the onset of the crisis. We show that banks that had a higher share of sovereign debt on their balance sheets were not necessarily operating riskier balance sheets. Seventh, we carry out some placebo exercises to convince the reader that the effects are indeed a feature of this particularly stressful period and are not confounded by other factors. The placebo exercises complement the first exercise that demonstrates parallel trends.

4 A Model of Investment, Leverage, and Debt Maturity

We use an arguably exogenous variation to identify an aggregate shock to the financial conditions that firms face. In doing this, we show distinct heterogeneous patterns in the responses of firms to the financial shock. While the interpretation of the aggregate shock is relatively straightforward, it is not so for the heterogeneous responses. For example, we cannot randomly assign firm characteristics such as leverage and maturity and, therefore, we require a model to interpret these findings.

As a benchmark, we use a standard model of investment with constant returns, idiosyncratic, uninsurable risk, and shocks to the interest rate to interpret our empirical results. Further, we extend the model by adding the choice of maturity. The analysis provides conditions for leverage and debt maturity to interact with the sensitivity of firms' investment decisions to interest rate shocks. We analyse both the cases in which the observed variation in debt maturity is "exogenous", and the more empirically relevant case in which the observed variation in leverage, debt maturity, and investment captures an omitted variable that jointly determines the latter two variables. In the second case, we interpret our empirical specification as capturing the reduced form relation between investment, the interest rate shock, leverage, and debt maturity.[12]

We find that the presence of long-term investment projects and frictions to the issuance of long-term debt, as captured by a firm-specific term premium, are needed for the model to account for the heterogeneous responses of investments to the financial shock in our empirical analysis. Thus, through the light of the theory, our empirical results highlight the importance of these model elements to understand the heterogeneous real effects of financial shocks.

4.1 Model economy

We study the problem of an entrepreneur who lives for three periods, owns a long-term project, and has access to an additional risky, linear investment opportunity in the interim period, $t = 1$. The entrepreneur finances the new investment and the possible negative cash flows associated with the long-term investment through the issuance of short- and long-term debts. The entrepreneur faces a credit shock in the interim period, that is, the cost of credit in this period is uncertain. Consumption takes place only in the last period, $t = 2$. As in Brunnermeier and Sannikov (2014) and Arellano et al. (2017), the key friction in the model is the inability of entrepreneurs to insure against idiosyncratic investment risk. We allow entrepreneurs to insure, at least partially, against the financial shock by managing the maturity of their debt.[13]

The entrepreneur starts the first period, $t = 0$, with a long-term project with deterministic cash flows $\{y_t\}_{t=0}^{2}$. Cash flows might be negative due to the initial investment or payments of previously issued debts. In the first period, the entrepreneur chooses short-term (1-period) and long-term (2-period) debts d_0^1 and d_0^2 (bond purchases if negative) to finance a given amount of

[12] In the Online Appendix we consider various extensions and modification of the benchmark model: (a) a richer specification in which the aggregate shock also affects the revenue of entrepreneurs; (b) an alternative model featuring risk-neutrality, diminishing returns, and collateral constraints, another common set of assumptions in the macro-finance literature; (c) a version of the model where initial leverage is endogenously determined by firms' productivity and initial net-worth. These results are discussed in Section 4.4

[13] We obtain an equivalent characterisation of the investment decision if we assume instead that the interim investment is limited by a collateral constraint and the return of the investment is deterministic but only if we maintain the assumption of constant returns, see the Online Appendix.

leverage d_0 subject to the budget constraint in the first period,[14]

$$d_0^1 + d_0^2 = d_0 = -y_0.$$

We denote r_0^1 and r_0^2 as the interest rates associated with the initial issuances of short- and long-term debts, respectively. At the beginning of the interim period, $t = 1$, a new value of the short-term interest rate $r_1^1 \in [\underline{r}, \bar{r}]$ is realised. In addition, in the interim period the entrepreneur has access to an investment opportunity k with an uncertain return $z \in [0, \infty)$. She can issue new debt d_1^1 to rollover the short-term debt issued in the first period or finance the new investment subject to the following budget constraint:

$$k = y_1 - \left(1 + r_0^1\right) d_0^1 + d_1^1.$$

In the final period, $t = 2$, the last cash flows from the long-term project occur, the return on the interim investment is realised, short- and long-term debts are repaid, and consumption is given by the last period's budget constraint:

$$c_2 = y_2 + zk - \left(1 + r_1^1\right) d_1^1 - \left(1 + r_0^2\right) d_0^2.$$

Consolidating the budget constraints of the three periods, the problem of the entrepreneur can be simplified to that of choosing the maturity of the debt in the initial period d_0^2 and the investment in the interim period k to maximise the expected utility of consumption in the final period, given log preferences[15]:

$$\begin{aligned}
&\max_{d_0^2, k} E_{r_1^1, z} \left[\log c_2\right] \\
&\text{s.t.} \\
&c_2 = \left(z - 1 - r_1^1\right) k + y_2 + \left(1 + r_1^1\right) \left(y_1 - \left(1 + r_0^1\right) d_0\right) \\
&+ \left(\left(1 + r_1^1\right) \left(1 + r_0^1\right) - \left(1 + r_0^2\right)\right) d_0^2.
\end{aligned} \tag{3}$$

In the analysis that follows we make two additional assumptions. First, we restrict the long-term interest rate, r_0^2, so that the net return of long-term debt is strictly negative (positive) in the lowest (highest) interest rate state:

[14]In referring to the total initial liabilities d_0 as leverage, we are implicitly assuming that the size of the initial long-term investment is common and equal to 1. It is relatively straightforward to endogenize the initial long-term investment by assuming a linear stochastic technology with returns in the interim and final period. The analysis of the investment decision in the interim period is unaffected if we assume that the uncertainty about the profile of returns of the long-term technology is realised at the beginning of the interim period.

[15]Most of the analysis generalises to the case of a general constant relative risk aversion (CRRA) preferences. In particular, the results in Propositions 1-4 generalise to the more general specification of preferences. The analysis of this case is available in Section C of the Online Appendix.

Assumption 1 *We assume that*

$$\left(1+r_0^1\right)(1+\underline{r})-1<r_0^2<\left(1+r_0^1\right)(1+\bar{r})-1. \tag{4}$$

As inspecting the consolidated budget constraint (3) shows, this assumption guarantees that long-term debt is an effective asset for transferring resources from low to high interest rate states.

In addition, we restrict the values of the initial leverage, the cash flows from the long-term project, and the interest rates in the first period to guarantee that the net worth in the interim period is positive for all values of $r_1^1 \in [\underline{r}, \bar{r}]$:

Assumption 2

$$y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}>0, \textit{ for all } r_1^1 \in [\underline{r}, \bar{r}].$$

This assumption guarantees that investment in the interim period is positive for all values of $r_1^1 \in [\underline{r}, \bar{r}]$. In addition, if there are financing needs in the interim period, that is, $y_1-(1+r_0^1)d_0<0$; then this assumption requires that the cash flows from the long-term project are strictly positive in the last period, $y_2>0$.

Related, the set of feasible values for the long-term debt are described by the following inequality:

$$y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2>0, \text{ for all } r_1^1 \in [\underline{r}, \bar{r}]. \tag{5}$$

The values of long-term debt in this set guarantee that the net-worth in the interim period is positive, and the investment is positive for all realisations of the interest rate, conditional on a given value of the maturity structure d_0^2. Given Assumption 2, $d_0^2=0$ is in this set.[16]

We first discuss the investment choice in the interim period, given leverage d_0 and the maturity structure in the first period $d_0^1=d_0-d_0^2$ and d_0^2, and then consider the maturity choice in the initial period.

[16]The implied set of feasible values of the long-term debt lie in the following interval

$$-\frac{y_2+(1+\bar{r})\left(y_1-\left(1+r_0^1\right)d_0\right)}{(1+\bar{r})\left(1+r_0^1\right)-\left(1+r_0^2\right)}<d_0^2<\frac{y_2+(1+\underline{r})\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left(1+r_0^2\right)-(1+\underline{r})\left(1+r_0^1\right)},$$

which is guaranteed to be non-empty by assumption 2.

4.2 Investment decision

The investment conditional on leverage, debt maturity, and the interest rate shock in the interim period is as follows:

$$
\begin{aligned}
& k\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \\
& = \bar{k}\left(r_1^1\right) \cdot \underbrace{\left[y_1 - \left(1 + r_0^1\right)\left(d_0 - d_0^2\right) + \frac{y_2 - \left(1 + r_0^2\right) d_0^2}{1 + r_1^1}\right]}_{\omega}. \qquad (6)
\end{aligned}
$$

The optimal investment decision can be expressed as product of two terms. The first term in the last line, $\bar{k}(r_1^1)$, captures the effect of the user cost of capital on the profitability of investment.[17] This term is a decreasing function of the cost of credit in the interim period, $\partial\bar{k}\left(r_1^1\right)/\partial r_1 < 0$. This derivative captures the pure effect of an interest rate shock on the net return on investment. Naturally, as the cost of credit increases, the net return on investment declines and, therefore, the desired investment decreases.[18] The second term, labelled ω, is the value of the net worth of the entrepreneur conditional on the realisation of the interest rate shock. These are the total resources available to invest. This term is independent of the interest rate shock when there are no discounted future cash flows that affect the net worth, i.e., $y_2 - (1 + r_0^2)d_0^2 = 0$.[19]

In our empirical analysis, we study the sensitivity of investment to a credit shock, which we demonstrate to be associated with a rise in the cost of credit. Furthermore, we show that leverage and the fraction of short-term debt amplify the effect of the credit shock. We now show that, when taking the decision on the debt maturity as exogenously given, this is a natural implication of the model if there are positive net future cash flows from the long-term investment.

The object of interest is the sensitivity of investment with respect to the interim interest rate that is obtained by log-differentiating equation (6) as follows:

[17] In particular, $\bar{k}(r_1^1)$ is implicitly defined by the following optimality condition

$$\mathbb{E}_z\left[\bar{k}\left(r_1^1\right) + \frac{1 + r_1^1}{z - 1 - r_1^1}\right]^{-1} = 0.$$

[18] Strictly speaking, this result depends on the log utility assumption. In general, an increase in r_1^1 will have an income and a substitution effect. With CRRA preferences, the absolute risk aversion is decreasing in income, implying that risk-taking increases with income. If $\bar{k}(r_1^1) > 1$, that is, if the entrepreneur is leveraged in the interim period, the income effect will reinforce the substitution effect and the result generalises to any CRRA utility function. If $\bar{k}(r_1^1) < 1$, the income effect will lead the entrepreneur to increase the fraction of the wealth invested in the risky investment. This effect will be relatively more important the higher the coefficient of relative risk aversion. In Section C of the Online Appendix, we further elaborate on this result.

[19] In the Online Appendix we derive an equivalent expression for the case with no investment risk but with limited commitment. In particular, we assume that the future cash flow of the long-term project can be perfectly collateralized while only a fraction, ϕ_z, of the investment in the interim period can be pledged. In this case, $\bar{k}(r_1^1) = (1 + r_1^1)/(1 + r_1^1 - \phi_z z)$.

$$\frac{\partial \log k}{\partial r_1^1} = \frac{1}{\bar{k}(r_1^1)} \frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} - \frac{1}{\omega} \frac{y_2 - (1 + r_0^2) d_0^2}{(1 + r_1^1)^2}. \tag{7}$$

The first term on the right-hand side captures the effect of the interest rate shock on the investment decision through its effect on the net return on investment. The second term is the effect of the interest rate on the investment decision through its effect on the net worth, ω, in the interim period. This second effect operates through the discounting of future cash flows, $y_2 - (1 + r_0^2) d_0^2$.

As stated in the following proposition, the elasticity of investment with respect to the interim interest rate decreases with total leverage if and only if the cash flows in the last period net of the long-term debt payments are positive, $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$.

Proposition 1 *If and only if* $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$ *then*

$$\frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} < 0.$$

The sensitivity of investment to the interest rate shock is only a function of leverage through its effect on the net worth in the interim period ω, as seen in the second term of the right hand side of equation (7). A higher leverage implies lower net-worth in the interim period and, thus a more negative sensitivity of investment to the interest rate shock when the discounted cash flows from the last period are positive, $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$.[20]

In addition, the elasticity of investment with respect to the interim interest rate increases with the amount of long-term debt given assumptions 1 and 2. This is stated in the following proposition.

Proposition 2 *If and only if* $y_2/(1 + r_0^2) + y_1/(1 + r_0^1) - d_0 > 0$ *then*

$$\frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} > 0.$$

Importantly, condition $y_2/(1 + r_0^2) + y_1/(1 + r_0^1) - d_0 > 0$ is implied by assumptions 1 and 2.

By increasing the amount of long-term debt the entrepreneur reassigns resources across periods and interest rate states. The financial needs in the interim period decline and so do the net cash flows in the last period. The net effect is positive as long as the long-term rate is not too large and the future cash flows are sufficiently high, as guaranteed by assumptions 1 and 2.

The condition in Proposition 1 is stronger than that in Proposition 2. To prove Proposition 2, we use assumptions 1 and 2. When the term premium is strictly positive: $1 + r_0^2 > \left(1 + r_0^1\right) \mathbb{E}\left(1 + r_1^1\right)$,

[20]Moreover, if there is a financial need in the interim period, that is, $y_1 - (1 + r_0^1)(d_0 - d_0^2) < 0$, the requirement that the net worth in the interim period is positive, that is, condition (5), means that there has to be positive net future cash flows from the long-term investment of $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$.

there are financing needs in the interim period, $y_1 - (1 + r_0^1)\left(d_0 - d_0^2\right) < 0$. In this case, the condition in Proposition 1 is implied by assumptions 1 and 2.

In addition, when there are financing needs in the interim period the effect of an increase in leverage on the elasticity of investment to the interim interest rate is greater (in absolute value) than that of the impact of an increase in long-term debt. This is shown in the following corollary to the previous two propositions:

Corollary 1 *If and only if* $y_1 - (1 + r_0^1)(d_0 - d_0^2) < 0$, *then*

$$\left| \frac{\partial^2 k\left(r_1^1\right)}{\partial r_1^1 \partial d_0} \right| > \left| \frac{\partial^2 k\left(r_1^1\right)}{\partial r_1^1 \partial d_0^2} \right| .$$

As we discussed before, a decline (negative increase) in the amount of long-term debt affects the sensitivity of investment by affecting the resources in the interim and the last period. The effect in the interim period is akin to an increase in leverage because it depletes the resources available to invest. The effect through the net cash flows in the last period is negative if there are financial needs in the interim period.

This last corollary highlights an interesting testable implication of the model, which is consistent with the empirical results presented in Table 5, which show that the magnitude of the effect of leverage is greater than that of the maturity structure of debt.

4.3 Maturity decision

The above analysis takes as given the maturity structure of the debt in the initial period. We now study the optimal choice of maturity and, therefore, how the maturity structure depends on the primitives of the model, such as the timing of the cash flows of the long-term investment, $\{y_t\}_{t=0}^2$, the term premium, $1 + r_0^2$, and the initial leverage, d_0. This analysis guides us to interpret the variation in the debt maturities observed in the data and our empirical results. In particular, we characterise the reduced form relation between investment, the interest rate shock, leverage, and debt maturity, when these variables partially capture omitted variables that jointly determine investment and debt maturity.[21]

Substituting the optimal investment decision in equation (6) into the expression for consumption in equation (3), the optimal debt maturity problem can be expressed as follows:

$$\max_{d_0^2} \mathbb{E}_{r_1^1} \left\{ \log \left[y_1 - \left(1 + r_0^1\right) d_0 + \frac{y_2}{1 + r_1^1} + \left(1 + r_0^1 - \frac{1 + r_0^2}{1 + r_1^1}\right) d_0^2 \right] \right\},$$

where we have rearranged the expression for the net worth in the interim period to highlight the return of the long-term debt, $1 + r_0^1 - (1 + r_0^2)/(1 + r_1^1)$.

[21] In our empirical analysis, we control for additional firm characteristics, such as measures of profitability, age, size, and location and industry fixed effects. In this analysis we assume that these controls are only imperfect measures of the timing of the cash flows from the long-term project or the time-zero interest rates.

The first order condition that characterises the optimal choice of debt maturity (see Appendix for details) is:

$$\mathbb{E}_{r_1^1}\left\{\frac{1+r_0^1-\frac{1+r_0^2}{1+r_1^1}}{y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2}\right\}=0. \tag{8}$$

The numerator inside the expectation is the return on long-term debt. This return increases with the interim period's interest rate. The return is weighted by the marginal utility of consumption that in the log case is the reciprocal of the net worth in the interim period.

We first consider the case in which the expectation hypothesis holds, i.e,

$$1+r_0^2=\left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right).$$

In this case, we obtain the following expression for the optimal debt maturity:

$$d_0^2=d_0-y_1/\left(1+r_0^1\right). \tag{9}$$

The long-term debt is chosen to finance all the initial leverage that cannot be paid back with the cash flows from the interim period. The variation in the amount of long-term debt, conditional on leverage, is driven solely by the variation in the cash flows in the interim period y_1.[22]

Solving for the short-term cash flows as a function of leverage and maturity, $y_1=(1+r_0^1)(d_0-d_0^2)$, and substituting into (6), we obtain a reduced form relation between investment, the interest rate shock, leverage, and debt maturity, which we assume to be the key variables that are observed in our empirical analysis[23]

$$\begin{aligned}\hat{k}\left(r_1^1,d_0,d_0^2\right)&=k\left(r_1^1,d_0,d_0^2,(1+r_0^1)(d_0-d_0^2),y_2,r_0^1,r_0^2\right)\\&=\bar{k}\left(r_1^1\right)\cdot\left[(1+r_0^1)(d_0-d_0^2)-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right]\\&=\bar{k}\left(r_1^1\right)\cdot\frac{1}{1+r_1^1}\left[y_2-\left(1+r_0^2\right)d_0^2\right].\end{aligned}$$

It follows that the (reduced-form) elasticity of investment with respect to the interest rate shock

[22]This is a knife-edge result specific to the case with log utility. With a general CRRA specification, it will be optimal to be more (less) hedged if the coefficient of risk aversion is greater (less) than 1. In Section C of the Online Appendix we elaborate on this result further.

[23]To simplify the analysis, we assume that there is no heterogeneity in the last period's cash flows, y_2, or the interest rates faced by the entrepreneur in the initial period, r_0^2. In the more general case, we would need to integrate with respect to these additional dimensions of heterogeneity.

is independent of the leverage and debt maturity:

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} = \frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} = 0.$$

In this case the intuition is relatively straightforward. When we observe an entrepreneur with higher leverage, holding fixed the optimal choice of long-term debt, her cash flows in the interim period y_1 must be larger, as follows from equation (9). This implies that the net worth in the interim period of this entrepreneur is unchanged and, therefore, the sensitivity of investment to the interest rate shock does not respond to the observed variation in leverage. A similar reasoning applies to the case in which we vary the maturity, while holding the leverage fixed.

A limitation of the case where the expectation hypothesis holds is that the cash flows in the last period do not affect the choice of maturity. This is no longer true in the more general case. We next consider a situation with a positive term premium, which is the empirically relevant case.

Given Assumption 2, it is straightforward to show that

$$\frac{\partial d_0^2}{\partial \left(1 + r_0^2\right)} < 0.$$

This means that entrepreneurs have financial needs in the intermediate period, that is, when $1 + r_0^2 > \left(1 + r_0^1\right) \mathbb{E} \left(1 + r_1^1\right)$, we have $y_1 - (1 + r_0^1)(d_0 - d_0^2) < 0$. In other words, the entrepreneurs have to rollover part of the initial debt in the interim period that further exposes them to the interest rate risk. Notwithstanding this, we show below that the reduced form sensitivity of investment to an interest rate shock is not affected by the leverage or the observed variation in the maturity structure, if this variation is driven by the timing of cash flows from either variation in y_1 or y_2.

As before, the quantity of long-term debt is a decreasing function of the cash flows in the interim period, but now the effect is stronger:

$$\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1 + r_0^1} = \left.\frac{\partial d_0^2}{\partial y_1}\right|_{1+r_0^2=\left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)}.$$

The stronger effect is explained by the fact that the demand for interest rate insurance is a decreasing function of the net worth when the utility function exhibits decreasing absolute risk aversion, for example, as is true in the case with CRRA preferences.[24]

This analysis indicates two important sources of variation in the maturities of debt. The first is given by the timing of the cash flows of the long-term investment, that is, variation in y_1 or y_2.

[24] Due to this effect, the amount of long-term debt issued is a decreasing function of the cash flows in the last period. Similarly, the effect of initial leverage on the amount of long-term debt issued is also stronger,

$$\frac{\partial d_0^2}{\partial y_1} = -\frac{1}{1 + r_0^1} - \frac{1 + r_0^2}{1 + r_0^1} \frac{\partial d_0^2}{\partial y_2} = -\frac{1}{1 + r_0^1} \frac{\partial d_0^2}{\partial d_0}.$$

The second is given by variation in the term premium across entrepreneurs, r_0^2, a simple reduced form way to capture idiosyncratic frictions to issuing long-term debt. As the previous analysis shows, entrepreneurs whose projects mature early or face a higher term premium choose to issue more short-term debt.

We now analyse the reduced form relation between investment, the interest rate shock, leverage, and debt maturity, $\hat{k}(r_1^1, d_0, d_0^2)$. To simplify the analysis, we consider the case in which there are only two dimensions of heterogeneity: leverage and either the timing of the cash flows from the long-term investment or the term premium, that is, either y_1, y_2, or r_0^2. In this case, the reduced form relation is implicitly defined by the following two equations:

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_1, y_2, r_0^2) \tag{10}$$

and

$$\mathbb{E}_{r_1^1}\left\{\frac{1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}}{y_1 - \left(1 + r_0^1\right) d_0 + \frac{y_2}{1+r_1^1} + \left(1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}\right) d_0^2}\right\} = 0. \tag{11}$$

The right-hand side of equation (10) is the optimal investment choice as a function of the primitives of the model given by equation (6) in which the implicit dependence of either y_1, y_2, or r_0^2 on d_0 and d_0^2, is defined by equation (11). Equation (11) just restates the first order condition with respect to the long-term debt in equation (8).

We next characterise the reduced form relation between investment, the interest rate shock, leverage, and debt maturity, $\hat{k}(r_1^1, d_0, d_0^2)$. As the following two propositions show, different sources of endogenous variation in the maturity of debt are associated with very different implications for the sensitivity of investment to interest rate shocks.

We first consider the case in which debt maturity, conditional on leverage, varies due to the heterogeneity across entrepreneurs in the timing of the cash flows from the long-term project, y_1 and y_2.

Proposition 3 *Assume* $1 + r_0^2 \geq \left(1 + r_0^1\right) \mathbb{E}\left(1 + r_1^1\right)$ *and that entrepreneurs are heterogeneous with respect to the initial leverage* d_0*, and either* y_1 *or* y_2*. Let* $\hat{k}(r_1^1, d_0, d_0^2)$ *be the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity defined implicitly by equations (10) and (11); then*

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} = \frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} = 0.$$

When the differences in the maturity structure of debt, conditional on leverage, are driven by differences in the cash flows from the long-term project, y_1 and y_2, the differential leverage (debt maturity), conditional on debt maturity (leverage), is not associated with a differential sensitivity of investment to the interest rate shock.

To see the intuition behind this result, consider two entrepreneurs with different levels of leverage and timing of the cash flows, but the same amount of long-term debt. In this case, the entrepreneur with the higher leverage must have higher cash flows in the first or second period so that the same maturity choice given by equation (11) is still optimal. If the same choice is rationalised by a higher cash flow in the first period, the result is straightforward. In this case, the higher value of y_1 that is required to keep the maturity constant exactly compensates for the larger value of the leverage and keeps the financing needs in the interim period unchanged. In the case where the cash flow in the last period vary, the reasoning is more subtle, as now the financing needs in the interim period are affected by the change in leverage, or $y_1 - (1 + r_0^1)(d_0 - d_0^2)$ is more negative. However, the lower cash flows in the last period mean that fewer future cash flows would need to be discounted as well. The fact that maturity is optimally chosen, that is, that equation (11) is satisfied, implies that these two effects cancel each other. A similar intuition applies when we consider two entrepreneurs with different amounts of long-term debt, but the same leverage.

It is important to highlight that this result holds true for the case in which the expectation hypothesis does not hold. In general, the optimal choice of debt maturity does not insulate the cash-flow, in the interim period, from the interest rate shock. The key message from this result is that firms with different choices of long-term debt, that are driven by differences in the timing of the cash-flows, are not differentially affected by the interest rate shock.

On the contrary, when the differences in the maturities of debt are driven by differences in the term premia that entrepreneurs face in the initial period, $1 + r_0^2$, the differential debt maturity is associated with a greater sensitivity of investment to the interest rate shock. This is established in the following proposition:

Proposition 4 *Assume that entrepreneurs are heterogeneous with respect to the initial leverage d_0 and the term premium r_0^2. Let $\hat{k}(r_1^1, d_0, d_0^2)$ be the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity defined implicitly by equations (10) and (11). Consider the case in which the entrepreneur has financing needs in the interim period: $\left(1 + r_0^1\right)\left(d_0 - d_0^2\right) - y_1 > 0$. Then,*

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} < 0$$

and

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} > 0.$$

This result is relatively straightforward. Consider two entrepreneurs with different levels of leverage and term premium, but that choose the same amount of long-term debt. Naturally, the entrepreneur with more leverage must be the one facing a higher cost of long-term borrowing. As shown in Proposition 1, the direct effect of having more leverage increases the sensitivity of investment to the interest rate shock. Furthermore, the higher cost of long-term debt further reduces the cash flows in the interim period reinforcing the direct effect. The intuition for the second part is similar. In this case, consider two entrepreneurs with the same amount of leverage, but with different amounts of long-term debt. The difference in the maturity structures is driven by the difference in the costs of long-term debt. The entrepreneur with more long-term debt is the one facing a lower cost. As shown in Proposition 2, the direct effect of having more long-term debt decreases the sensitivity of investment to the interest rate shock. Furthermore, the lower cost of long-term debt further improves the cash flows in the interim period that reinforces the direct effect.

These results, together with our empirical analysis, indicate that it is important to model frictions to the issuance of long-term debt to account for the effects of the financial crisis on firms' investment, beyond the friction in the baseline model of the imperfect insurance with respect to idiosyncratic investment risk. In our model, frictions to the issuance of long-term debt can be captured by a firm-specific term premium.

4.4 Discussion of alternative assumptions

In this section, we explore the sensitivity of the results to various extensions and modifications of the benchmark model. We provide a brief summary of the results below and relegate the detailed analysis to the Online Appendix (see sections D, E, and F).

Richer Aggregate Shock We consider an extension of the benchmark model where the interest rate shock in the interim period has a negative effect on the cash flows from the long-term project, $y_1(r_1^1)$, $\partial y_1(r_1^1)/\partial r_1^1 < 0$. This extension captures the case in which the financial crisis may have demand spillovers that affect the profits of entrepreneurs beyond the direct effect through their cost of financing.

In this case, the sensitivity of investment to the interest shock is more negative, as the shock has an added negative effect on the cash flows in the interim period that further depletes the resources available to invest, as shown by the following expression that generalises equation (7):

$$\frac{\partial \log k\left(r_1^1\right)}{\partial r_1^1} = \frac{1}{\bar{k}(r_1^1)} \frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} - \frac{y_2 - (1+r_0^2)d_0^2}{\omega} \frac{1}{(1+r_1^1)^2} + \frac{1}{\omega} \frac{1}{1+r_1^1} \frac{\partial y_1\left(r_1^1\right)}{\partial r_1^1}. \tag{12}$$

In turn, this extra negative effect allows for a stronger version of Propositions 1 and 2. That is, when there is an exogenous increase in leverage or an exogenous decline in the maturity of debt, investment is more sensitive to the interest rate shock under weaker assumptions.[25]

However, the results in Proposition 3 are strengthened. In this extension we find that if maturity is endogenously determined by the variation in the cash flows from the long-term project, y_1 and

[25] In particular, Proposition 1 now requires the weaker condition $y_2 - (1 - r_0^2)d_0^2 - \partial y_1(r_1^1)/\partial r_1^1 > 0$. Proposition 2 now requires $y_2/(1+r_0^2) + y_1/(1+r_0^1) - d_0 - ((1+r_1^1)/(1+r_0^2) - 1/(1+r_0^1))\partial y_1(r_1^1)/\partial r_1^1 > 0$. The second condition is weaker if we consider a relatively high realisation of the interest shock, $(1+r_1^1)(1+r_0^1) - (1+r_0^2) > 0$, as in a financial crisis.

y_2, then when we observe an entrepreneur with higher leverage or lower debt maturity, holding fixed either their maturity or leverage, we should find that the investment of this entrepreneur is less sensitive to the interest rate shock.[26] When we observe individuals with high leverage or a short debt maturity, holding fixed debt maturity or leverage, we are selecting individuals with higher cash flows from the long-term project, y_1 or y_2. The indirect selection effect counterbalances the negative direct effects. Thus, in this extension the countervailing selection effect is larger and, hence, more than compensates for the stronger direct negative effect. In terms of the second term on the right-hand side of equation (12), i.e., $(y_2 - (1+r_0^2)d_0^2)/\omega$, the results in Proposition 3 were about the direct and indirect effects cancelling out. The new effects operate through the last term of equation (12): $1/\omega$. Importantly, when differentiating the added term, there will be one less negative indirect effect on the sensitivity, which indicates that the overall net effect is positive.

Limited Commitment, Risk Neutrality, and Diminishing Returns We consider an alternative set of assumptions that are also common in the macro-finance literature: limited commitment, risk neutrality, and diminishing returns (Khan and Thomas, 2013; Buera et al., 2015). We modify the benchmark model by introducing these assumptions sequentially.

First, we obtain an equivalent characterisation of the investment decision if we abstract from investment risk and, instead, assume that the interim investment is limited by a collateral constraint, provided that we maintain the assumption of constant returns. In particular, we get exactly the same characterisation if the debt payments in the final period are constrained by:

$$\left(1+r_0^2\right) d_0^2 + \left(1+r_1^1\right)\left(k + \left(1+r_0^1\right)\left(d_0 - d_0^2\right) - y_1\right) \leq y_2 + \phi_z z k + \phi_k \left(1-\delta\right) k,$$

in which $d_1^1 = k + \left(1+r_0^1\right)\left(d_0 - d_0^2\right) - y_1$ is the debt issued in the interim period, and ϕ_z and ϕ_k are the fractions of the return to interim investment and the capital stock that can be recovered in the event of default. In this extension, we also allow for a partial depreciation of capital $\delta \in [0,1]$ (which was set to 1 in the benchmark model). We obtain a similar characterisation of the investment decision in the interim period as a linear function of the net worth in which $\bar{k}(r_1^1) = (1+r_1^1)/(1+r_1^1 - \phi_z z - \phi_k(1-\delta))$.

Second, if we assume risk neutrality in the modified version of the model with limited commitment, we do not obtain a bounded solution for the maturity choice. For instance, if the expectation hypothesis holds, $E_{r_1^1}\left[(1+r)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right] = 0$, the optimal choice is to set d_0^2 to $-\infty$. In this case, by buying (an arbitrarily large quantity of) long-term bonds the entrepreneur transfers resources to the low interest rate states in which the investment to net worth ratio, $(1+r_1^1)/\left(1+r_1^1 - \phi_z z - \phi_k(1-\delta)\right)$, is the largest. Thus, to have a well-defined choice of maturity,

[26] We still get the result in Proposition 3 only for the case in which the heterogeneity of maturities is driven by variations in the cash flows in the interim period, y_1, and we consider the effect of leverage on the reduced form investment relation: $\partial^2\hat{k}/(\partial r_1^1 \partial d_0) = 0$. In all the other cases described in Proposition 3, we find that investment is less sensitive to the interest rate shock when leverage increases or the maturity of debt declines: $\partial^2\hat{k}/(\partial r_1^1 \partial d_0) > 0$, $\partial^2\hat{k}/(\partial r_1^1 \partial d_0^2) < 0$.

we require an alternative source of concavity.[27]

Next, we add diminishing returns to the investment in the interim period. To obtain a closed form solution for the constrained level of investment, we assume $\phi_z = 0$. With this assumption, conditional on the entrepreneur being constrained, we obtain a similar characterisation of the investment decision in the interim period as a linear function of the net worth in which $\bar{k}(r_1^1) = (1 + r_1^1)/(1 + r_1^1 - \phi_k (1 - \delta))$. Therefore, exact counterparts to Propositions 1 and 2 are true in this extension if we are considering a value of r_1^1 for which the entrepreneur is constrained. The main difference is that there could be realisations of the interest rate shock for which the entrepreneur is unconstrained. In these states of the world, trivially, the sensitivity of investment is independent of (exogenous) changes in leverage and debt maturity.

If we assume that there are financial needs in the interim period, $(1 + r_0^1)(d_0 - d_0^2) - y_1 > 0$, and that the net cash flows from the long-term project in the final period are positive, $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$, then there is a maximum realisation of the interest rate that supports a strictly positive investment. These assumptions guarantee that there will be a high marginal value of net worth in high interest rate states. This is a force that leads the entrepreneur to choose a positive amount of long-term debt, as in the benchmark model.[28]

Importantly, in this extension Proposition 3 holds. In particular, we find that if maturity is endogenously determined by the variation in the cash flows from the long-term project, y_1 and y_2, then when we observe an entrepreneur with higher leverage or a shorter debt maturity, holding fixed either their maturity or leverage, we should find that the investment of this entrepreneur is not differentially sensitive to the interest rate shock. It is straightforward to show that a counterpart of Proposition 4 holds for this economy if the optimal amount of long-term debt is a decreasing function of its cost, or r_0^2. Thus, all the results in the benchmark model extend to this economy with risk neutral investors and diminishing returns to the investment in the interim period with the caveat that because of the diminishing return to the investment in the interim period, the entrepreneur could be unconstrained in some states of the world.

Endogenous Leverage The last extension that we consider is a version of the model in which initial leverage is endogenously determined by the firms' productivity and initial resources in the first period. As in the previous extension, we perform the analysis in a model that features diminishing returns and collateral constraints and is extended to have a short-term investment decision in the first period, but abstracting from the choice of maturity to focus on the endogenous determination of leverage. The analysis in this case is done for the case without cash flows from a pre-existing long-term project. Given these assumptions, the constrained level of investment is

[27]Notice that in the case of risk neutrality we do not need Assumption 2, as now negative cash flows in the final period are feasible.

[28]For this result we also require that $\underline{r}$ is not too low. For sufficiently low values of $\underline{r}$, the unconstrained level of capital is arbitrarily larger than the amount that is feasible with the limited commitment constraint. In this case, the marginal value of resources in the low interest rate states is also particularly high and choosing a negative value for the long-term debt (positive amount of long-term assets) might be optimal.

independent of the realisation of the interest rate. This independence allows us to focus on the roles of the initial net worth and the productivity in the initial period.

In this framework, the investment of constrained entrepreneurs does not respond to an interest rate shock if the collateral constraint is not affected by the shock. In contrast, the investment of unconstrained entrepreneurs is a decreasing function of the interest rate. We also show that the relation between the initial leverage and the future constrained state of an entrepreneur depends crucially on the heterogeneity in the initial leverage. In this case, if the heterogeneity in the initial net worth drives the initial leverage, entrepreneurs with higher initial leverage are more likely to be constrained and, therefore, the sensitivity of the entrepreneurs' investment to the interest rate shock is a decreasing function of leverage. The opposite result is obtained when the variation in initial leverage is driven by the heterogeneity in the initial productivities of entrepreneurs. In this case, entrepreneurs that are highly leveraged in the first period are more likely to have realised high productivity and, therefore, to have a larger net worth in the interim period. If productivities across periods are independent, this net worth means that the entrepreneurs are less likely to be constrained in the interim period and, as a result, are more sensitive to the interest rate shock. Again, for this characterisation it is important that the constrained level of investment is not a function of the interest rate shock, for example, there are no future cash flows to be discounted. These results echo, in a more stylised framework, the recent numerical findings by Ottonello and Winberry (2018).

5 Empirical Determinants of Debt Maturity

A key result of the model is that the maturity structure only matters if it is determined by the heterogeneity in the term premium and not cash flows. We use our data and try to provide more direct evidence in favour of this result. We do this in two steps. First, we show that the maturity structure is determined, at least in part, by term premia. Second, we show that the heterogeneity in responses of investment to the financial shock is in fact driven by the term premium component. For the first part, we estimate an equation of the form:

$$(LT\_debt\_share)_{i,t} = f\left(X_{i,t}\right),$$

in which the left-hand side represents the long-term debt as a fraction of total debt for firm 'i' at time 't'. The vector $X_{i,t}$ contains a set of firm-specific characteristics that include variables such as firm-specific borrowing costs, cash flows, investment, and external finance dependence.[29] The goal here is not to make causal statements but to explore which of the variables are most closely related with the long-term debt issuance of a firm, focusing mainly on cash flows and firm firm-specific debt costs. We do not have information on firm-specific interest rates at different levels of maturity

[29] Calculated at a sectoral level following the methodology developed in Rajan and Zingales (1998). It is defined as $(capital\_expenditure - cash\_flows)/capital\_expenditure$.

and, therefore, we construct a broad proxy for debt costs. From the accounting database, we have information on total interest paid by firms, and we use this information to construct our measure of funding costs as the total interest expenditure normalised by total debt. The cash flows are also normalised by total debt. Investment is defined as the growth rate of fixed assets.

Table 6 presents the results. We use data from 2009-2014 in columns 1 - 4. In columns 5 - 8, we use the cross section Q4:2009 - Q4:2010. The results are in alignment with our theoretical results, as the cost of funds and the cash flows show up with statistically significant and negative signs across all specifications. However, we need to understand how important each of these variables is economically. In terms of economic magnitudes, a one standard deviation increase in the cost of funds results in a decline in the share of long-term debt by approximately 23 p.p. (column 1), while a one standard deviation increase in cash flows leads to a reduction in the share of long-term debt by approximately 6 p.p (column 2). This reduction indicates that the heterogeneity in the issuance of long-term debt might be driven, predominantly, by debt costs rather than cash flows. As mentioned earlier, we do not wish to make causal claims here. We simply wish to explore which of the variables are most closely related with the issuance of long-term debt of a firm.

For the second part, we recreate columns 5 - 8 of Table 5 but explicitly control for funding costs. The findings are reported in Table 7. The firm level sovereign exposures are now also interacted with a dummy that equals one if the firm is in the top quartile of the cost of funds (i.e., paying an average interest rate of greater than 6.15%). We include this interaction simultaneously with the interaction with the maturity structure. In the table, we observe that the inclusion of the interaction of the financial shock with cost of funds makes the interaction with the maturity structure weaker. These results provide evidence that shows the heterogeneity in responses to the financial shock by maturity structure is present when maturity is driven by the term premium component.[30]

6 Conclusion

Using a rich loan-level dataset from Portugal, we study how a financial shock is transmitted to the real sector. We first analyse the effects on the credit supply and then study the firms' performance in terms of employment, assets, liabilities, and usage of intermediate commodities in the aftermath of the sovereign debt crisis. We identify two important dimensions of firm heterogeneity. Specifically, we show that *ex ante* highly leveraged firms and firms with a shorter maturity of debt experience sharper contractions in credit and were unable to tap into alternative sources of funding. The credit supply contraction also had real effects. The same firms that experienced a reduction in credit also contracted significantly more than their counterparts. In addition to performing the analysis by comparing the most leveraged firms and the firms with the highest fraction of short-term debt (top quartile) with their counterparts, we also study the effects

[30]We have also tested a specification in which we added an interaction between the financial shock and the cash flows. Consistent with the theory, the presence of the cash-flow interaction does not attenuate the estimates for the maturity structure heterogeneity.

along the entire distribution of leverage and debt maturity.

We also present a model of investment and debt maturity with uninsurable investment risk. We use the model to study the effect of a credit shock and, therefore, to interpret our empirical results. The model highlights the conditions under which leverage and debt maturity are key factors that determine the sensitivity of firms' investment to a credit shock. In addition, the model provides a theory that sheds light on the determinants of the maturity of a firm's debt vis-a-vis our empirical results. We show that when differences in the maturity structure of debt are driven by the heterogeneity in the maturities of investment projects (i.e., heterogeneity in cash flows from projects), a higher quantity of short-term debt is not associated with a higher sensitivity of investment to credit shocks. On the contrary, when the differences in the maturity of debt are driven by heterogeneity in the term premia faced by the firm, a higher quantity of short-term debt is associated with a higher sensitivity of investment to shocks. Furthermore, we use our data and provide more direct empirical evidence in favour of this result.

We show that the results in this model generalise to alternative specifications of financial frictions, preferences, and technologies that are also common in the macro-finance literature. Notwithstanding this generalisation, one limitation of the theory is that the analysis is done in a three-period environment. We conjecture that the key lessons would apply to an infinite horizon framework if the maturity structure could be continuously adjusted to hedge for future interest rate shocks. At the same time, a richer multi-period environment with costs to the adjustment of the maturity of debt would provide a natural micro-foundation for the firm-specific term premium discussed in Proposition 4. We see this as a promising avenue for future research.

The policy implications are straightforward. The leverage of non-financial firms remain high by historical standards, and they remain an important source of vulnerability for the outlook of the corporate sector. Our results show that, besides the overall amount of debt, the strength of a firm in terms of income generation and the maturity composition of the debt are important determinants of firms' performance during a crisis episode. Moreover, a larger share of short-term debt increases corporate vulnerabilities as it exposes firms to rollover risk more frequently. Our analysis shows that it is important to understand the drivers of a shorter maturity of debt. When framing policies, heterogeneities along the dimensions such as cash flow generation and firm-specific debt costs must be taken into consideration.

References

Acharya, V., T. Eisert, C. Eufinger, and C. Hirsch (2018): "Real Effects of the Sovereign Debt Crisis in Europe: Evidence from Syndicated Loans," *Review of Financial Studies*, 31(8), 2855–2896.

Almeida, H., M. Campello, B. Laranjeira, and S. Weisbenner (2012): "Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis," *Critical Finance Review*, 1, 3–58.

Angeletos, G. (2007): "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," *Review of Economic Dynamics*, 10, 1–30.

Arellano, C., Y. Bai, and L. Bocola (2017): "Sovereign Default Risk and Firm Heterogeneity," *NBER Working Paper No. 23314*.

Arellano, C., Y. Bai, and P. Kehoe (2016): "Financial Frictions and Fluctuations in Volatility," *Minneapolis Fed Staff Report 466*.

Banco de Portugal (2019a): "Bank Balance Sheet Database (CBSD)," *DOI: https://doi.org/10.17900/BBS.Jun2021.V1*.

——— (2019b): "Central Balance Sheet Database (CBSD)," *DOI: https://doi.org/10.17900/CB.CBA.Jun2021.V1*.

——— (2019c): "Central Credit Responsibility â Exposure Level (CRC)," *DOI: https://doi.org/10.17900/CRC.EXP.Jun2019.V1*.

Benmelech, E., C. Frydman, and D. Papanikolaou (2018): "Financial Frictions and Employment during the Great Depressions," *Journal of Financial Economics*.

Bentolila, S., M. Jansen, G. Jimenez, and S. Ruano (2018): "When Credit dries up: Job losses in the Great Recession," *Journal of the European Economic Association*, 16(3), 650–695.

Bloomberg L.P. (2021): "Credit Default Swap and Security Information," Accessed 31-August-2021.

Bofondi, M., L. Carpinelli, and E. Sette (2018): "Credit Supply during a Sovereign Debt crisis," *Journal of the European Economic Association*, 16(3), 696–729.

Bottero, M., S. Lenzu, and F. Mezzanotti (2015): "Sovereign debt exposure and the bank lending channel: impact on credit supply and the real economy," *Bank of Italy, Economic Research and International Relations Area*.

Brunnermeier, M, K. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *The American Economic Review*, 104, 379–421.

Brunnermeier, M., L. Garicano, R. Lane, P, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. Van Nieuwerburgh, and D. Vayanos (2011): "European Safe Bonds (Esbies)," *www.euronomics.com*.

Buera, F., R. Fattal-Jaef, and Y. Shin (2015): "Anatomy of a credit crunch: From Capital to Labor Markets," *Review of Economic Dynamics*, 18, 101–117.

Chodorow-Reich, G. (2014): "The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis," *The Quarterly Journal of Economics*, 129, 1–59.

Cingano, F., F. Manaresi, and E. Sette (2016): "Does Credit Crunch Investments Down? New Evidence on the Real Effects of the Bank-Lending Channel," *The Review of Financial Studies*, 29, 2737–2773.

Crouzet, N. and N. R. Mehrotra (2018): "Small and Large Firms Over the Business Cycle," Working Papers 18-09, Center for Economic Studies, U.S. Census Bureau.

Davis, H. and J. Haltiwanger (1992): "Gross Job Creation, Job Destruction and Employment Reallocation," *The Quarterly Journal of Economics*, 107, 819–863.

De-Socio, A. and E. Sette (2018): "Firms? investments during two crises," Temi di discussione (Economic working papers) 1173, Bank of Italy, Economic Research and International Relations Area.

European Central Bank (2016): "Euro area and national MFI interest rates (MIR)," http://www.ecb.europa.eu/stats/money/interest/interest/html/index.en.html.

Gertler, M. and S. Gilchrist (1994): "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms," *The Quarterly Journal of Economics*, 109, 309–340.

Giroud, X. and M. Mueller, H (2017): "Firm Leverage, Consumer Demand, and Employment Losses During the Great Recession," *The Quarterly Journal of Economics*, 132, 271–316.

Holmstrom, B. and J. Tirole (1997): "Financial Intermediation, Loanable Funds, and the Real Sector," *The Quarterly Journal of Economics*, 112, 663–691.

Iyer, R., J. Peydro, S. Lopes, and A. Schoar (2014): "Interbank Liquidity Crunch and the Firm Credit Crunch: Evidence from the 2007-2009 Crisis," *The Review of Financial Studies*, 27, 347–372.

Kalemli-Ozcan, S., L. Laeven, and D. Moreno (2020): "Debt Overhang, Rollover Risk, and Corporate Investment: Evidence from the European Crisis," Tech. rep., University of Maryland.

Khan, A. and J. Thomas (2013): "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Journal of Political Economy*, 121, 1055 – 1107.

Khwaja, A., I. and A. Mian (2008): "Tracing the Impact of Bank Liquidity Shocks: Evidence from an Emerging Market," *The American Economic Review*, 1413–1442.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Ottonello, P. and T. Winberry (2018): "Financial Heterogeneity and the Investment Channel of Monetary Policy," *NBER Working Paper No. 24221*.

Rajan, R. G. and L. Zingales (1998): "Financial Dependence and Growth," *American Economic Review*, 88, 559–586.

A Tables

Variables	CBSD Mean	CBSD SD	CBSD & CRC Mean	CBSD & CRC SD	¿1 Relations Mean	¿1 Relations SD
Employment	13.66	120.34	21.56	149.42	25.33	162.57
Fixed Assets	934068.3	2.98e+07	1761126	4.06e+07	2094338	4.54e+07
Tot. Liab	2848650	8.58e+07	5572900	1.49e+08	6719576	1.67e+08
Int. Comm. Usage	203245.3	2.05e+06	325180.60	2.6e+06	390843.58	3.01e+06
EBIT	80525.3	2684130	137002	3605431	168671.72	4045453
Age	12.22	11.84	13.79	12.34	14.72	12.49
No. of firms	138639		106723		82561	

Variable	High Leverage Mean	High Leverage SD	Low Leverage Mean	Low Leverage SD	High ST Debt Mean	High ST Debt SD	Low ST Debt Mean	Low ST Debt SD
Employment	29.73	245.50	23.58	114.17	23.26	200.58	27.07	121.64
Fixed Assets	4756188	8.29e+07	1055956	1.36e+07	890535.7	7903117	3109972	6.12e+07
Tot. Liab	1.77e+07	3.12e+08	2381914	1.80e+07	3041880	3.65e+07	9818381	2.24e+08
Int. Comm.	559478.1	4868151	324254.7	1786554	346758	3310647	429390.3	2701344
EBIT	305742.9	7122610	114449.7	1662172	102505.3	1908473	224422.1	5203882
Age	14.35	12.88	14.85	12.32	14.14	12.31	15.19	12.61

Table 1: Descriptive Statistics (Firms)

Note: The tables above show the firm characteristics from different perspectives. The firm characteristics reported are employment, fixed assets, total liabilities, intermediate commodity usage, earnings before interest and taxes, and age. All figures reported correspond to Q4:09. CBSD is the firm balance sheet data, CRC is the central credit registry. The left most panel on the top table shows the firms that file taxes, the central panel shows all firms that file taxes and have lending relationships with one or more banks, while the right most panel shows those firms that have relationships with multiple banks (our focus). The bottom panel further zooms in on firms in the top right panel and helps us shed some light on firm characteristics based on their leverage and maturity structure of debt. High leverage corresponds to leverage above 47%, while high short-term debt corresponds to short-term debt above 53%.

Variables	All Banks Mean	SD	High Sov Share Mean	SD	Low Sov Share Mean	SD	P Value (t-test)
Total Assets (bn)	14.1	28.3	18.3	35.2	11.5	21.4	0.44
Capital Ratio	14.85	7.74	15.17	8.80	14.59	6.98	0.83
Liquidity Ratio	13.44	15.96	16.54	17.08	10.87	14.97	0.31
Overdue/total loans	2.72	2.62	2.91	2.86	2.57	2.51	0.71
Corp. Share	28.84	18.73	27.90	15.01	30.41	21.65	0.59
Hhs. Share	25.59	23.55	19.84	14.55	30.39	28.56	0.20
Funding (securities/assets)	6.32	9.74	7.05	10.62	4.91	8.70	0.45
Funding (inter-bank/assets)	24.46	19.78	25.00	21.54	24.01	18.28	0.88
Funding (central bank/assets)	7.49	13.98	9.71	16.27	6.65	11.92	0.41
Loan to deposit	2.22	2.24	1.88	1.59	2.50	2.68	0.43
No. of banking groups	33		15		18		

Table 2: Descriptive Statistics (Banks)

Note: Figures are for Q4:09. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share greater than 6%. Overdue/total loans is a measure of risk on the banks' balance sheet. We next report the share of bank lending going to the corporate and the household sectors. Funding from securities is a measure of market dependence of the bank. We also report funding obtained from the interbank market and the central bank, as fractions of total assets. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

Variables	High Sov Share Mean	SD	Low Sov Share Mean	SD	P Value
Age	19.24	4.73	18.79	5.01	0.79
Firmsize	15.32	0.78	15.68	0.91	0.24
ST debt share	0.27	0.09	0.23	0.09	0.21
Leverage	0.62	0.24	0.79	0.32	0.13
Profitability	0.01	0.01	0.01	0.05	0.75
NPL ratio	0.02	0.01	0.03	0.05	0.57
No. of banking groups	15		18		

Table 3: Banks' Weighted Borrower Characteristics

Note: Figures are for Q4:09. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share greater than 6%. The figures above correspond to weighted average borrower characteristics of each bank. The weights are calculated using outstanding loans as of Q4:09. Firmsize is the log of total assets, ST debt share is short-term debt normalised by total debt, leverage is defined as all interest bearing liabilities normalised by total assets, profitability is earnings before interest and taxes normalised by total assets, and NPL ratio is the non-performing loans as a fraction of total loans. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

	(1) Leverage $n > 1$	(2) ST Debt $n > 1$	(3) Leverage $n > 1$	(4) ST Debt $n > 1$	(5) Lev + ST Debt $n > 1$	(6) Lev $n \geq 1$	(7) ST Debt $n \geq 1$	(8) Lev + ST Debt $n \geq 1$
Sov_share	0.139	0.218	0.350	0.442	0.477	0.280	0.392	0.435
	(0.433)	(0.413)	(0.499)	(0.493)	(0.488)	(0.391)	(0.410)	(0.403)
Sov_share*Highlev	-0.356**		-0.334*		-0.330*	-0.309**		-0.359***
	(0.155)		(0.163)		(0.176)	(0.145)		(0.135)
Sov_share*ST Debt		-0.492***		-0.507**	-0.502**		-0.576**	-0.567**
		(0.181)		(0.204)	(0.204)		(0.222)	(0.222)
Cap_ratio			0.187	0.186	0.188	0.058	0.077	0.079
			(0.462)	(0.461)	(0.461)	(0.466)	(0.476)	(0.474)
Liq_ratio			1.155	1.220	1.223	0.929	0.897	0.899
			(1.169)	(1.165)	(1.166)	(1.122)	(1.170)	(1.167)
Bank_size			0.042**	0.044**	0.044**	0.033**	0.034**	0.034**
			(0.018)	(0.018)	(0.018)	(0.016)	(0.017)	(0.017)
NPL_share			-0.044	-0.012	-0.038	-0.037	-0.011	-0.035
			(0.098)	(0.097)	(0.096)	(0.097)	(0.096)	(0.096)
Highlev						0.028**		0.031***
						(0.011)		(0.011)
ST Debt							0.006	0.005
							(0.015)	(0.015)
Constant	-0.110***	-0.109***	-0.536***	-0.552***	-0.552***	-0.420**	-0.437**	-0.441**
	(0.030)	(0.031)	(0.186)	(0.187)	(0.187)	(0.185)	(0.189)	(0.188)
Firm FE	Y	Y	Y	Y	Y	N	N	N
Bank Controls	N	N	Y	Y	Y	Y	Y	Y
Observations	144,966	139,821	144,966	139,821	139,821	198,708	184,416	184,416
R-squared	0.440	0.424	0.444	0.429	0.429	0.004	0.005	0.005

Table 4: Lending Effects

Note: The dependant variable is the loan growth rate at the bank-firm level. Columns 1 - 5 represent regression results for firms having multiple banking relationships *ex ante*. Columns 1 and 2 present the baseline regressions (including interactions with leverage and short-term debt dummy) with firm fixed effects. Columns 3 and 4 introduce an array of bank level controls including capital ratio, liquidity ratio, size and non-performing loan share as a fraction of total loans. Column 5 includes both the leverage and short-term debt interactions in the same specification. Columns 6-8 include firms having single banking relationships as well. Clustered standard errors (bank level) are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Leverage Gr_emp	(2) Leverage Gr_ast	(3) Leverage Gr_liab	(4) Leverage Gr_int	(5) ST Debt Gr_emp	(6) ST Debt Gr_ast	(7) ST Debt Gr_liab	(8) ST Debt Gr_int	(9) Both Gr_emp	(10) Both Gr_ast	(11) Both Gr_liab	(12) Both Gr_int
Wtd_sov_holding	0.030	-0.279	0.233	0.024	0.017	-0.039	0.097	-0.019	0.047	-0.250	0.877**	0.050
	(0.083)	(0.248)	(0.206)	(0.078)	(0.090)	(0.256)	(0.349)	(0.092)	(0.084)	(0.238)	(0.356)	(0.078)
Wtd_sov_holding*Highlev	-0.199*	-0.834***	-1.605***	-0.450***					-0.194*	-0.825***	-2.408***	-0.443***
	(0.112)	(0.207)	(0.410)	(0.142)					(0.111)	(0.206)	(0.519)	(0.142)
Wtd_sov_holding* High_stdebt					-0.140**	-0.265**	-0.289**	-0.218***	-0.131*	-0.229**	-0.163	-0.199***
					(0.069)	(0.110)	(0.125)	(0.046)	(0.067)	(0.107)	(0.110)	(0.045)
Highlev	0.023***	-0.009	0.001	0.050					0.024***	-0.008	0.031	0.051
	(0.008)	(0.161)	(0.027)	(0.085)					(0.008)	(0.161)	(0.021)	(0.085)
High_stdebt					-0.023	-0.144	0.097***	0.000	-0.025	-0.290	0.133***	0.015
					(0.017)	(0.160)	(0.036)	(0.044)	(0.019)	(0.216)	(0.024)	(0.034)
Firm & Bank Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector and Location FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.031	0.081	0.033	0.053	0.031	0.082	0.016	0.052	0.031	0.081	0.026	0.053
Observations	88,204	89,410	89,466	89,823	88,204	89,410	89,828	89,823	88,204	89,410	89,828	89,823

Table 5: Real Effects: Interactions with leverage and short-term debt

Note: The dependant variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, between Q4:2009 and Q4:2010. These are indicated at the top of the columns. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in Q4:2009. Firm level controls include age, size, profitability, and sector and location fixed effects. Weighted bank controls include size, capital ratio, liquidity ratio, and average interest rates charged by the respective banks. These controls have been included in all specifications. Columns 1 - 4 present results for the interaction of the sovereign exposures with firm leverage, columns 5 - 8 present results for the interaction with short-term debt, while columns 9 - 12 report regressions where we include both interactions. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

VARIABLES	(1) CoF	(2) Cash	(3) Investment	(4) All	(5) CoF	(6) Cash	(7) Investment	(8) All
Cost of funds (CoF)	-0.359***			-0.244***	-0.288***			-0.141***
	(0.007)			(0.009)	(0.010)			(0.011)
Cash flow		-0.032***		-0.025***		-0.038***		-0.034***
		(0.001)		(0.001)		(0.001)		(0.001)
Investment			0.025***	0.020***			0.015***	0.015***
			(0.001)	(0.001)			(0.003)	(0.003)
Ext. dependence				0.019				0.023***
				(0.024)				(0.009)
Firm FE	Y	Y	Y	Y	N	N	N	N
Time FE	Y	Y	Y	Y	N	N	N	N
Observations	552,844	552,844	514,663	552,844	74,636	74,636	70,011	70,011
R-squared	0.590	0.591	0.586	0.589	0.032	0.047	0.023	0.047

Table 6: LT debt, Cost of funds, & Cash flow

Note: We use information on years 2009-2014 in columns 1- 4. In columns 5 - 8, we use the cross section Q4:2009 - Q4:2010. The dependent variable is the long-term debt as a fraction of total debt for a firm at any given time 't'. We introduce one variable at a time to note the correlations with the maturity structure. *** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.274***	-0.228	0.587	0.268***
	(0.100)	(0.250)	(0.437)	(0.092)
Wtd_sov_holding*High_cof	-0.649***	-0.654***	-0.323**	-0.773***
	(0.095)	(0.148)	(0.138)	(0.088)
Wtd_sov_holding*High_stdebt	-0.029	-0.075	-0.241*	-0.125**
	(0.072)	(0.092)	(0.187)	(0.048)
High_cof	-0.100***	-0.171***	-1.420***	-0.043
	(0.024)	(0.048)	(0.099)	(0.030)
High_stdebt	-0.031	-0.143	0.097	0.000
	(0.020)	(0.156)	(0.030)	(0.044)
Observations	77,188	78,085	78,412	78,411
R-squared	0.033	0.080	0.041	0.057

Table 7: Maturity Structure, Cost of funds, & Cash flow

Note: The dependant variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in Q4:2009. Firm level controls include age, size, profitability, and sector and location fixed effects. Weighted bank controls include size, capital ratio, liquidity ratio, and average interest rates charged by the respective banks. These have been included in all specifications. *** p<0.01, ** p<0.05, * p<0.1

B Figures

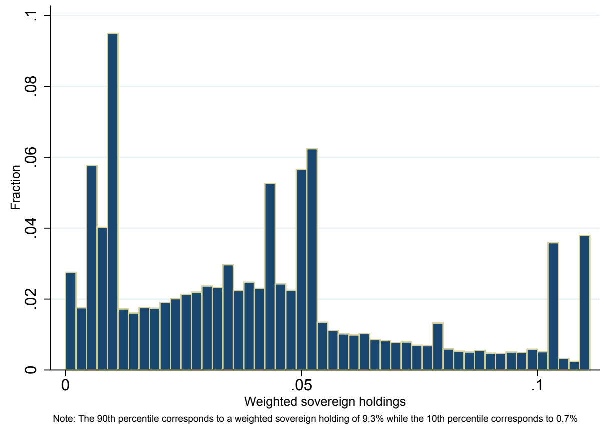


Figure 2: Firms' weighted sovereign exposures

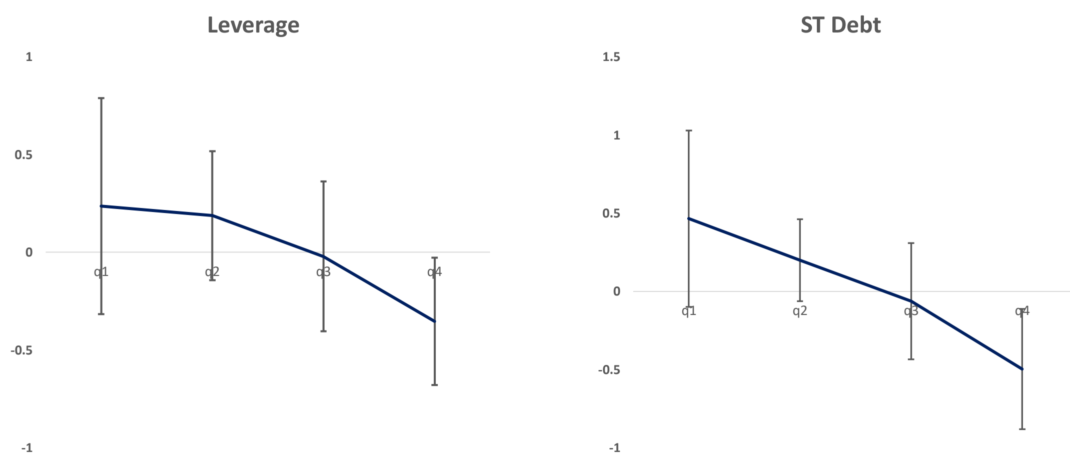
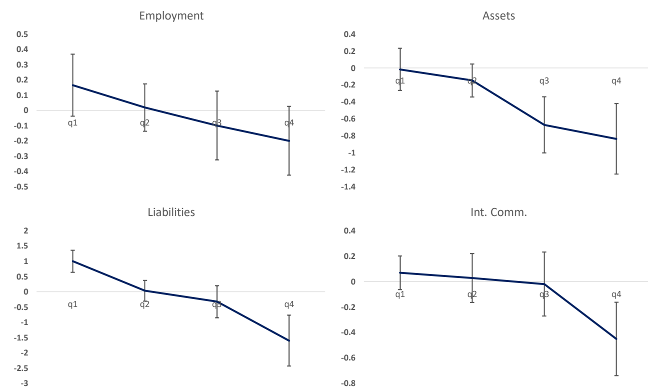
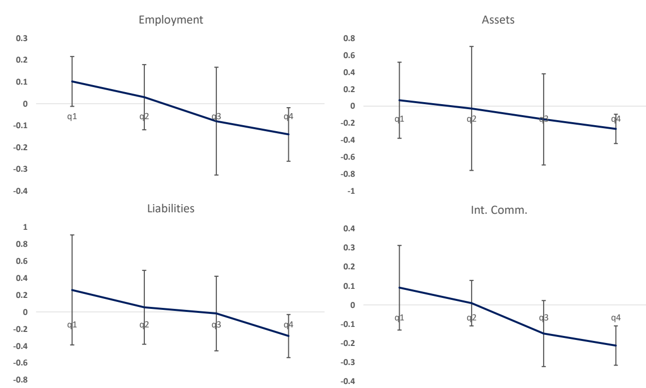


Figure 3: Lending effects by the respective quartiles



(a) Leverage



(b) ST Debt

Figure 4: Real effects by the respective quartiles

C Theoretical Appendix

In this appendix we characterise the model presented in Section 4 and provide the proofs of the propositions stated in that section.

Given the total leverage d_0 and the quantity of long-term debt d_0^2, the investment decision in the interim period solves

$$\max_k \mathbb{E}_z \{\log c_2\}$$

where

$$\begin{aligned} c_2 &= \left(z - 1 - r_1^1\right) k + y_2 + \left(1 + r_1^1\right) y_1 \\ &\quad - \left(1 + r_1^1\right)\left(1 + r_0^1\right)\left(d_0 - d_0^2\right) - \left(1 + r_0^2\right) d_0^2. \end{aligned}$$

The first-order condition is:

$$\mathbb{E}_z \left\{ \frac{z - 1 - r_1^1}{c_2} \right\} = 0.$$

The solution is given by

$$\begin{aligned} & k\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \\ & = \bar{k}\left(r_1^1\right) \cdot \left[y_1 - \left(1 + r_0^1\right) d_0 + \frac{y_2}{1 + r_1^1} + \left(1 + r_0^1 - \frac{1 + r_0^2}{1 + r_1^1}\right) d_0^2 \right] \\ & = \bar{k}\left(r_1^1\right) \cdot \omega \end{aligned}$$

where $\bar{k}\left(r_1^1\right)$ solves

$$\mathbb{E}_z \left[\frac{1}{\bar{k}\left(r_1^1\right) + \frac{1}{\frac{z}{1+r_1^1} - 1}} \right] = 0$$

with

$$\frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} = - \frac{\mathbb{E}_z \left[\frac{\frac{1}{\left(\frac{z}{1+r_1^1} - 1\right)^2} \frac{z}{\left(1+r_1^1\right)^2}}{\left(\bar{k}\left(r_1^1\right) + \frac{1}{\frac{z}{1+r_1^1} - 1}\right)^2} \right]}{\mathbb{E}_z \left[\frac{1}{\left(\bar{k}\left(r_1^1\right) + \frac{1}{\frac{z}{1+r_1^1} - 1}\right)^2} \right]} < 0,$$

and ω is the value of the net worth of the entrepreneur at the beginning of the intermediate period.

The semi-elasticity of investment with respect to the interest rate in the interim period is

$$\frac{\partial \log k\left(r_1^1\right)}{\partial r_1^1} = \frac{1}{\bar{k}(r_1^1)}\frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} - \frac{1}{\omega}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}. \tag{13}$$

The proof of propositions 1 and 2 follow from differentiating this expression with respect to leverage and the maturity of the debt in the first period.

Proof of Proposition 1: Differentiating (13) with respect to leverage

$$\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0} = -\frac{1+r_0^1}{1+r_1^1}\frac{1}{\omega}\frac{\frac{y_2-(1+r_0^2)d_0^2}{1+r_1^1}}{\omega} < 0.$$

The inequality follows from the condition $y_2-(1+r_0^2)d_0^2 > 0.\square$

Proof of Proposition 2: Differentiating (13) with respect to the maturity of the debt,

$$\begin{aligned}\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0^2} =& \frac{1}{\omega^2\left(1+r_1^1\right)^2} \\ & \left[\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)\left(y_2-(1+r_0^2)d_0^2\right)\right. \\ & \left. +\ (1+r_0^2)\left(y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right)\right] \\ =& \frac{(1+r_0^1)\left(1+r_0^2\right)}{\omega^2\left(1+r_1^1\right)^2}\left[\frac{y_2}{1+r_0^2}+\frac{y_1}{1+r_0^1}-d_0\right] > 0. \end{aligned} \tag{14}$$

where the inequality follows from assumptions 1 and 2.

Proof of Corollary 1: Rearranging equation (14)

$$\begin{aligned}\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0^2} =& \frac{1}{\omega^2\left(1+r_1^1\right)^2}\left(1+r_0^1\right)\left(1+r_0^2\right)\left(\frac{y_2}{1+r_0^2}-d_0^2\right) \\ & +\frac{1}{\omega^2\left(1+r_1^1\right)^2}\left(1+r_0^2\right)\left(1+r_0^1\right)\left(\frac{y_1}{1+r_0^1}-\left(d_0-d_0^2\right)\right) \\ =& -\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0} \\ & +\frac{1}{\omega^2\left(1+r_1^1\right)^2}\left(1+r_0^2\right)\left(1+r_0^1\right)\left(\frac{y_1}{1+r_0^1}-\left(d_0-d_0^2\right)\right).\end{aligned}$$

$\square$

C.1 Maturity decision

Using the optimal investment decision, consumption in the last period can be written as

$$
\begin{aligned}
c_2 &= (z-1-r_1^1)k\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) + y_2 - (1+r_0^2)d_0^2 + (1+r_1^1)(y_1 - (1+r_0^1)(d_0 - d_0^2)) \\
&= \left[(z-1-r_1^1)\bar{k}(r_1^1) + 1 + r_1^1\right]\left[y_1 - \left(1+r_0^1\right)d_0 + \frac{y_2}{1+r_1^1} + \left(1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right].
\end{aligned}
$$

Given the investment decision in the interim period, the optimal debt maturity solves

$$
\max_{d^2} \mathbb{E}_{r_1^1} \log\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]
$$

with first-order condition

$$
\mathbb{E}_{r_1^1} \frac{\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)}{y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2} = 0. \tag{15}
$$

When the expectation hypothesis holds, i.e., $1 + r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, $d_0^2 = d_0 - y_1/\left(1+r_0^1\right)$ solves the first-order condition,

$$
\begin{aligned}
\mathbb{E}_{r_1^1} \frac{\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - \left(1+r_0^2\right)d_0} &= \frac{\mathbb{E}_{r_1^1}\left[\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right]}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - \left(1+r_0^2\right)d_0} \\
&= 0.
\end{aligned}
$$

Assumption 2 implies that the amount of long-term debt is a decreasing function of the term premium

$$
\frac{\partial d_0^2}{\partial\left(1+r_0^2\right)} = -\frac{\mathbb{E}_{r_1^1}\left\{\frac{y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right)}{\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)^2}{\left[\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}} < 0. \tag{16}
$$

Next, we consider the comparative statics of long-term debt when there is a strictly positive term premium $1 + r_0^2 > \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$.

As before, the amount of long-term debt is a decreasing function of the cash flow in the interim period

$$
\begin{aligned}
\frac{\partial d_0^2}{\partial y_1} &= -\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(1+r_1^1\right)\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}} \\
&= -\frac{1}{1+r_0^1}\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2+\left(1+r_0^2\right)\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}} \\
&= -\frac{1}{1+r_0^1} \\
&\quad -\frac{1+r_0^2}{1+r_0^1}\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}} \\
&< 0.
\end{aligned}
$$

The first term equals the effect of y_1 on d_0^2 when the entrepreneur is not exposed to interest rate risk. As the cash flow in the interim period increases, more of the initial leverage can be repaid in one period and, therefore, less long-term debt needs to be issued. The sign of the second terms follows from (15) and the fact that when $1+r_0^2 > \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, which implies that $d_0^2 < d_0 - y_1/\left(1+r_0^1\right)$ and, therefore, that the net worth in the interim period, $y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)\left(d_0-d_0^2\right)\right)-\left(1+r_0^2\right)d_0^2$, is a decreasing function of r_1^1. The second term captures the effect of changes in the net worth on the demand for insurance. In general, the sign of this term depends on the coefficient of risk aversion. In our log case, the coefficient of absolute risk aversion is a strictly decreasing function of net worth. Therefore, the second term is negative.

All in all, when the term premium is positive, the result is a greater sensitivity of the long-term issuance to the cash flow in the interim period

$$
\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1+r_0^1} = \left.\frac{\partial d_0^2}{\partial y_1}\right|_{1+r_0^2=\left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)}.
$$

Related, the amount of long-term debt is a decreasing function of the cash flow in the last period y_2

$$\frac{\partial d_0^2}{\partial y_2} = -\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}} < 0.$$

As was the case when considering the effect of the cash flow in the interim period, as the coefficient of risk aversion is decreasing, the demand for insurance is a decreasing function of the cash flow in the last period.

We are now ready to prove Proposition 3.

Proof of Proposition 3: First, we consider the case in which entrepreneurs are heterogeneous with respect to the initial leverage and the income in the interim period y_1. Equation (11) defines implicitly a function relating y_1 and d_0 and d_0^2, which, abusing notation, we denote $y_1(d_0, d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_1(d_0, d_0^2)), \tag{17}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_2, r_0^1, r_0^2. Applying the Chain Rule on equation (17) and the Implicit Function Theorem to equation (11),

$$\begin{aligned}\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_1}\frac{\partial y_1}{\partial d_0^2} \\ =& \frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\ &- \frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(1+\tilde{r}_1^1\right)\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.\end{aligned}$$

Rearranging

$$\begin{aligned}=&\frac{\frac{1}{\omega^2}\frac{1}{(1+r_1^1)^2}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(1+\tilde{r}_1^1\right)\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\ &\left(1+r_0^2\right)\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]}\right\} \\ =&0,\end{aligned}$$

where the last equality uses the first-order condition for the optimal maturity choice, i.e., equation (15). Similarly,

$$\begin{aligned}\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_1}\frac{\partial y_1}{\partial d_0} \\ =& -\frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2-\left(1+r_0^2\right)d_0^2}{(1+r_1^1)^2} \\ &+\frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\left(1+r_0^1\right) \\ =&0.\end{aligned}$$

We next consider the case in which entrepreneurs are heterogeneous with respect to the initial leverage and the income in the final period y_2. Equation (11) defines implicitly a function relating y_2 and d_0 and d_0^2, which, abusing notation, we denote $y_2(d_0, d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_2(d_0, d_0^2)), \tag{18}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_1, r_0^1, r_0^2. Applying the Chain Rule on equation (19) and the Implicit Function Theorem to equation (11),

$$\begin{aligned}\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_2}\frac{\partial y_2}{\partial d_0^2} \\ =& \frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2-\left(1+r_0^2\right)\left(d_0-\frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\ &-\left[\frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\frac{1}{1+r_1^1}-\frac{1}{\omega}\frac{1}{(1+r_1^1)^2}\right] \\ &\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.\end{aligned}$$

Rearranging

$$\begin{aligned}=&\frac{1}{\omega^2}\frac{1}{(1+r_1^1)^2}\frac{\left(1+r_0^1\right)}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\ &\mathbb{E}_{\tilde{r}_1^1}\left[\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]}\right] \\ =&0.\end{aligned}$$

where, as before, the last equality uses the first-order condition for the optimal maturity choice, i.e., equation (15). Similarly,

$$\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_2}\frac{\partial y_2}{\partial d_0} \\
=& -\frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2-\left(1+r_0^2\right)d_0^2}{(1+r_1^1)^2} \\
&+\left[\frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\frac{1}{1+r_1^1}-\frac{1}{\omega}\frac{1}{(1+r_1^1)^2}\right] \\
&\left(1+r_0^1\right)\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(1+\tilde{r}_1^1\right)\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}$$

Rearranging

$$\begin{aligned}
=&\frac{\left(1+r_0^1\right)}{\omega^2}\frac{1}{(1+r_1^1)^2}\frac{1}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\
&\mathbb{E}_{\tilde{r}_1^1}\left[\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right. \\
&\left.\left[-y_2+\left(1+r_0^2\right)d_0^2+\left(1+\tilde{r}_1^1\right)\left(-y_1+\left(1+r_0^1\right)\left(d_0-d_0^2\right)\right)\right]\right] \\
=&\frac{\left(1+r_0^1\right)}{\omega^2}\frac{1}{(1+r_1^1)^2}\frac{1}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\
&\mathbb{E}_{\tilde{r}_1^1}\left[\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2}\right] \\
=&0.
\end{aligned}$$

□

Proof of Proposition 4: When entrepreneurs are heterogeneous with respect to the initial leverage and the term premium r_0^2, equation (11) defines implicitly a function relating r_0^2 and d_0 and d_0^2, which, abusing notation, we denote $r_0^2(d_0, d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, r_0^2(d_0, d_0^2)), \tag{19}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_1, y_2, r_0^1. Applying the Chain Rule on equation (19) and the Implicit Function

Theorem to equation (11),

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial r_0^2} \frac{\partial r_0^2}{\partial d_0^2} \\
=& \frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\
&+ \left[\frac{1}{\omega^2} \frac{\left(y_2 - (1+r_0^2) d_0^2\right) d_0^2}{(1+r_1^1)^2} - \frac{1}{\omega} \frac{d_0^2}{(1+r_1^1)^2}\right] \\
&\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
=& \frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\
&- \frac{\left(1+r_1^1\right)}{\omega^2} \frac{\left(y_1 - \left(1+r_0^1\right)\left(d_0 - d_0^2\right)\right) d_0^2}{(1+r_1^1)^2} \\
&\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} > 0,
\end{aligned}
$$

where the inequality follows from condition (2) and the assumption that the entrepreneur is leveraged in the interim period, i.e., $y_1 - \left(1+r_0^1\right)\left(d_0 - d_0^2\right) < 0$. Similarly,

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} + \frac{\partial^2 \log k}{\partial r_1^1 \partial r_0^2} \frac{\partial r_0^2}{\partial d_0} \\
=& - \frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right) d_0^2}{(1+r_1^1)^2} \\
&- \left[\frac{1}{\omega^2} \frac{\left(y_2 - (1+r_0^2) d_0^2\right) d_0^2}{(1+r_1^1)^2} - \frac{1}{\omega} \frac{d_0^2}{(1+r_1^1)^2}\right] \\
&\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
&= -\frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2-\left(1+r_0^2\right)d_0^2}{(1+r_1^1)^2} \\
&\quad+\frac{\left(1+r_1^1\right)}{\omega^2}\frac{\left(y_1-\left(1+r_0^1\right)\left(d_0-d_0^2\right)\right)d_0^2}{(1+r_1^1)^2} \\
&\quad\frac{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{\tilde{r}_1^1}\left\{\frac{y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+\tilde{r}_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+\tilde{r}_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} < 0,
\end{aligned}
$$

where the inequality follows from Assumptions 1 and 2, and the condition guaranteeing that the entrepreneur is leverage in the interim period, i.e., $y_1-\left(1+r_0^1\right)\left(d_0-d_0^2\right)<0$. □